

*NKS-R and NKS-B Joint Summary Seminar Armémuseum,  
Stockholm, 26th - 27th March 2009*

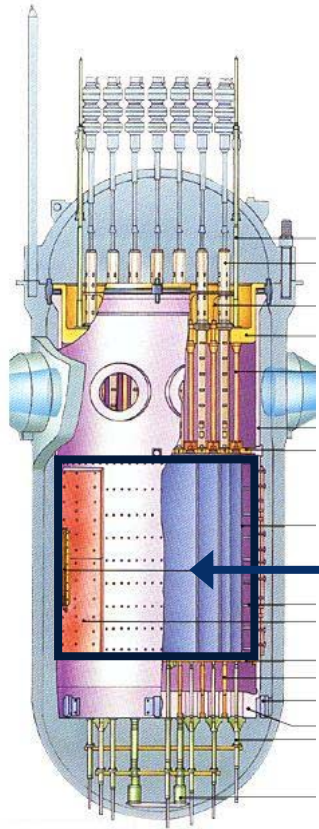
**IACIP: NKS\_R\_2008\_61:**

**Improving accuracy of the calculation of  
in-core power distributions for light water  
reactors**

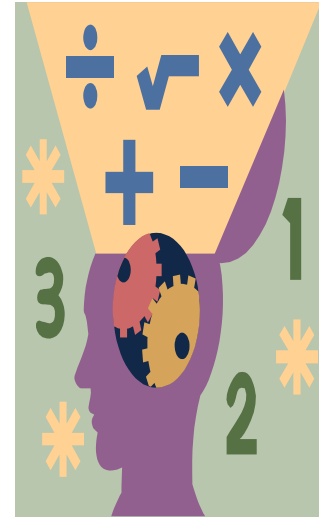
*Makoto Tsuiki and William H. Beere  
IFE, OECD Halden Reactor Project*



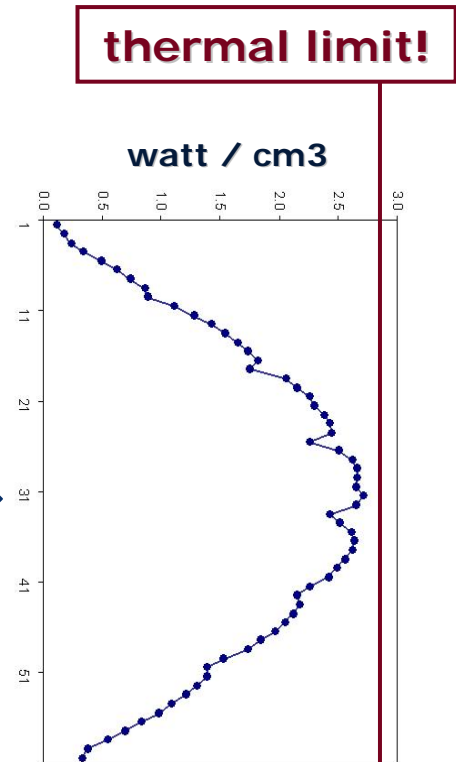
# LWR core neutronics calculation



core / fuel composition,  
geometry



mathematics model  
on neutrons  
behavior



power  
distribution,  
reactivity

# purpose

increase accuracy of power distribution calculations



less safety margin  
requirements



more economy oriented  
core operation

e.g., less fresh fuel  
higher fuel burnup  
less high level wastes



more accurate  
transient analysis



safer design of  
fuel and core

# basic equations

## static multigroup neutron transport equations

$$\begin{aligned} & \Omega \cdot \nabla \Phi_g(\mathbf{r}, \Omega) + \Sigma_g(\mathbf{r}) \Phi_g(\mathbf{r}, \Omega) = Q_g(\mathbf{r}) \\ \equiv & \int_{4\pi} d\Omega' \Sigma_{g'} \Sigma^s(\mathbf{r}; g, \Omega \leftarrow g', \Omega') \Phi_{g'}(\mathbf{r}, \Omega') \\ & + (\chi_g / 4\pi / k_{\text{eff}}) \int_{4\pi} d\Omega' \Sigma_{g'} v \Sigma_{f, g'}(\mathbf{r}) \Phi_{g'}(\mathbf{r}, \Omega') \end{aligned}$$

- $\Phi_g(\mathbf{r}, \Omega)$  : neutron angular flux (unknown function)  
 $k_{\text{eff}}$  : effective multiplication factor (unknown)  
 $\mathbf{r}$  : spatial position  
 $\Omega$  : direction of neutron motion  
 $g$  : kinetic energy of neutrons

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$$\Phi_g(\mathbf{r}, \Omega) = v n(\mathbf{r}, \Omega) [\text{cm} / \text{sec}][\text{neutrons} / \text{cm}^3]$$

# solution

power density distribution (energy / volume / time):

$$P(\mathbf{r}) = c \int_{4\pi} d\Omega \sum_g \Sigma_{f,g}(\mathbf{r}) \Phi_g(\mathbf{r}, \Omega)$$

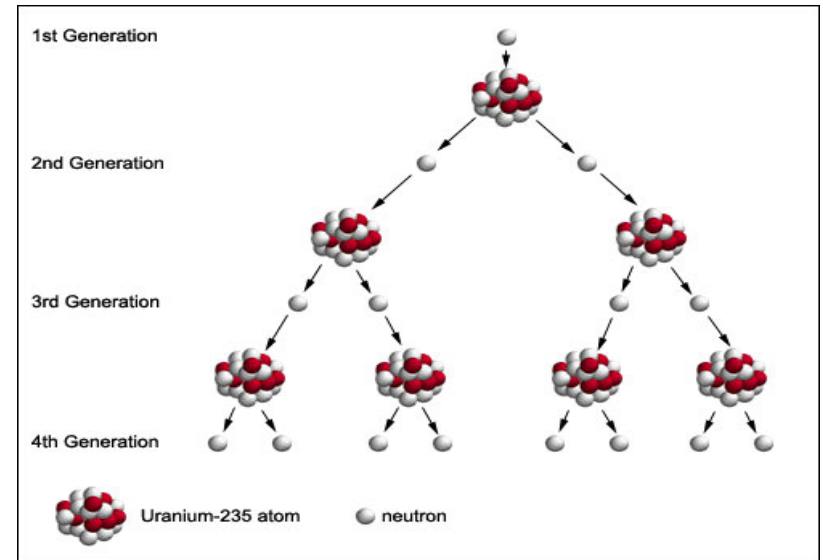
$c$  : energy release per fission

**keff**: core criticality

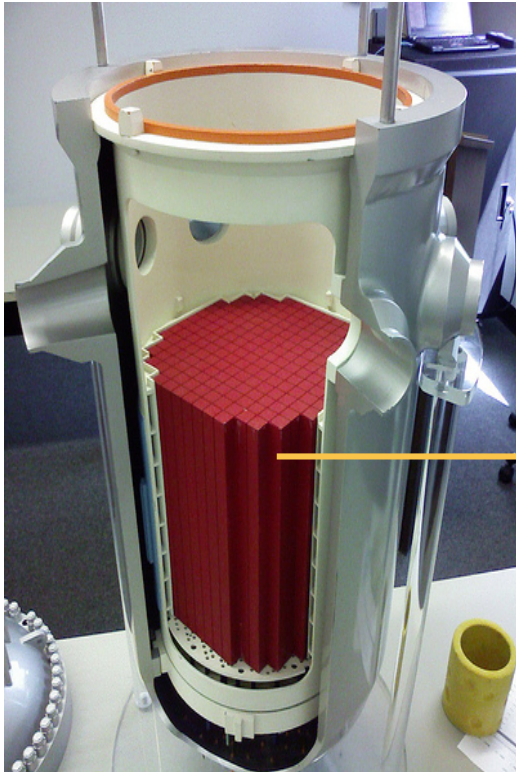
$keff > 1$ : supercritical

$keff = 1$ : critical

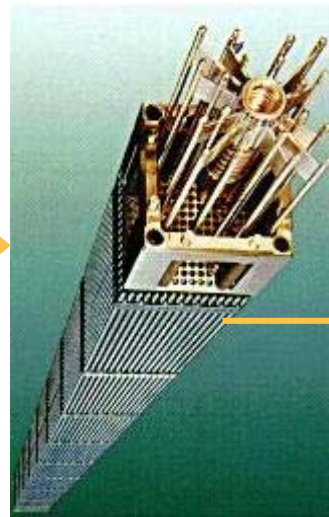
$keff < 1$ : subcritical



# difficulty (1) in space

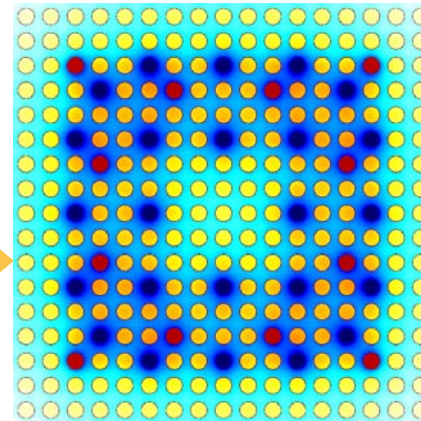


reactor core

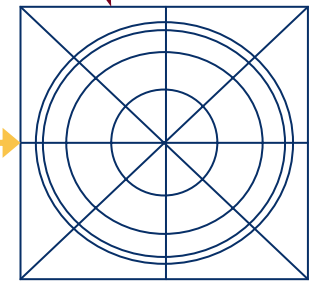


fuel assembly

very large number of regions  
with different material  
compositions



fuel rods



fuel cell



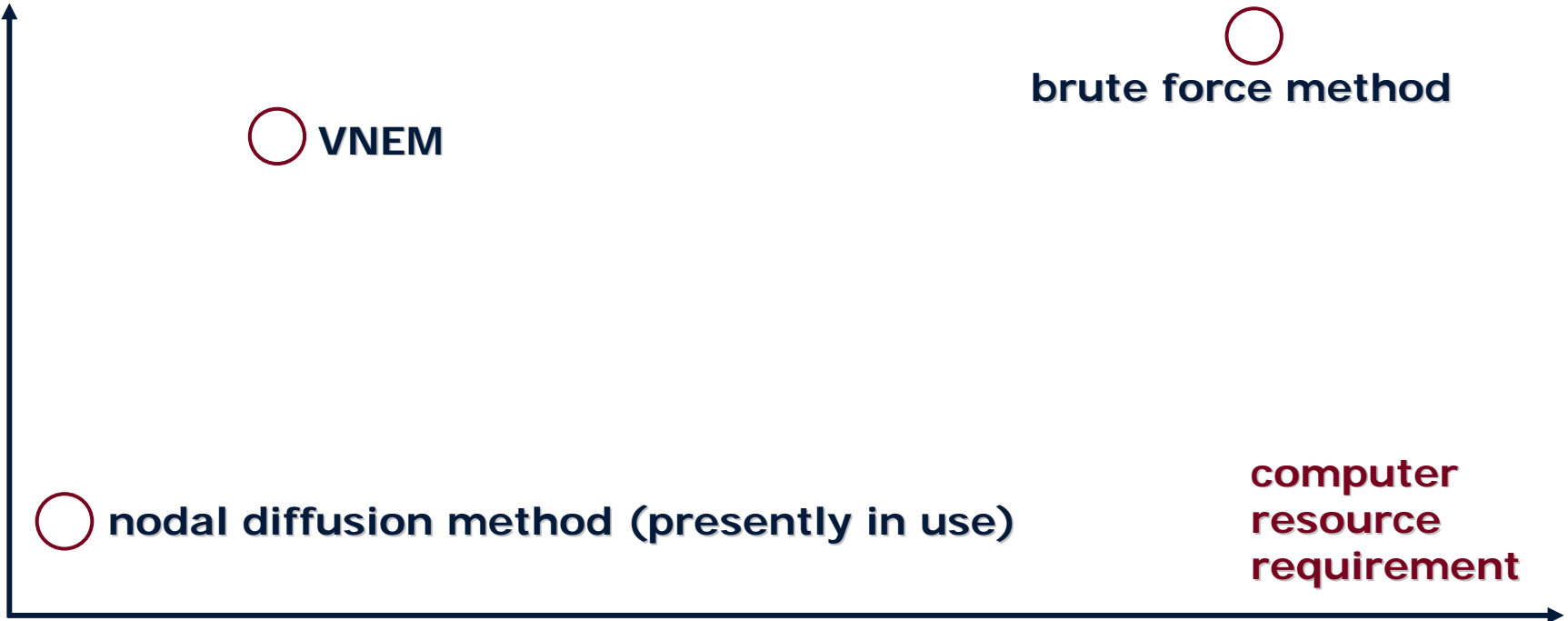
# conventional "nodal diffusion" methods

- 2 stages method: separate into assembly and full core
- ? diffusion approximation:  $\Omega$ -dependence almost ignored
- ? assembly homogenization: heterogeneity of intra-assembly structure
- energy group condensation



# our target

accuracy



# VNEM\* solution

higher accuracy :

solve rigorous equation

use even/odd PL transport theory

reduced computing resource :

- nodal expansion method
- variational (Ritz) method

\*Variational  
Nodal  
Expansion  
Method

# NEACRP PWR MOX benchmark

## participating transport codes

organization	code	method
CEN-MOL	DOT4.3	SN
ECN-PETTEN	DORT	SN
NFI	TWODANT	SN
PSI	BOXER	TPM
JAERI	TWOTRAN II	SN
	GMVP	MC
IKE	ICM2D	J+ -
ANL	VIM	MC
AEC/AEK	RED-1	BC
	RED-2	BC
<i>IFE</i>	<i>FCM2D</i>	<i>CM</i>
	<i>VNEM2D</i>	<i>VNEM</i>



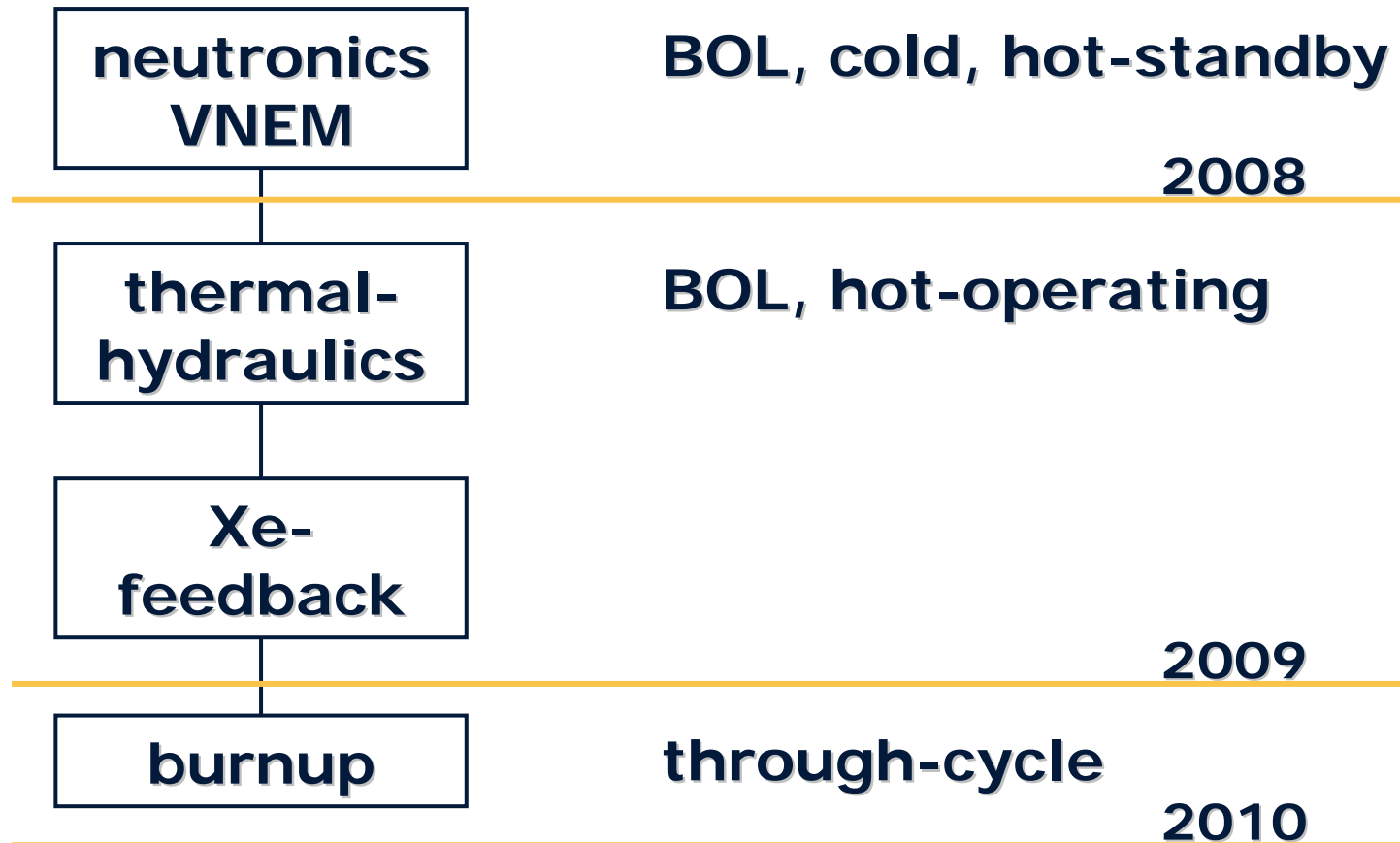
## keff comparison, case C3 (inner core)

organization	code	error pcm
CEN-MOL	DOT4.3	-101
ECN-PETTEN	DORT	-68
NFI	TWODANT	-121
PSI	BOXER	-112
JAERI	TWOTRAN II	-63
	GMVP	-111
IKE	ICM2D	-89
ANL	VIM	-136
AEC/AEK	RED-1	-90
	RED-2	-109
<i>IFE</i>	<i>FCM2D</i>	<i>1</i>
	<i>VNEM2D</i>	<i>-30</i>

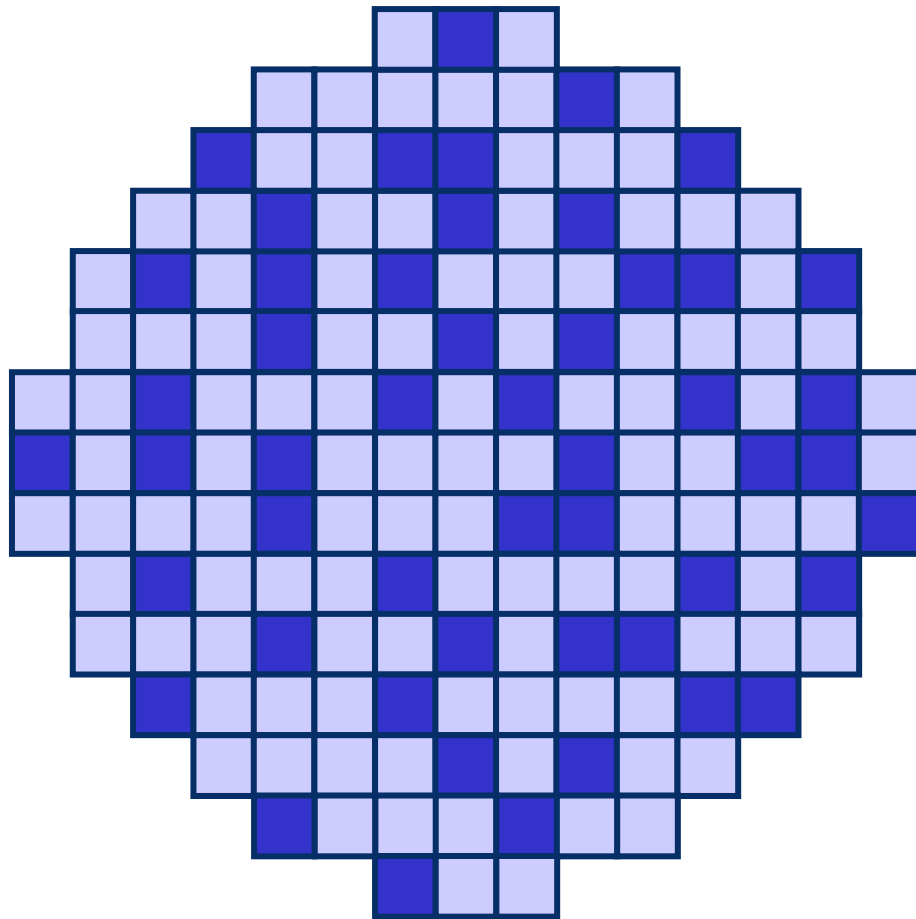
## keff comparison, case C5 (peripheral)

organization	code	error pcm
CEN-MOL	DOT4.3	-138
ECN-PETTEN	DORT	42
NFI	TWODANT	-8
PSI	BOXER	1
JAERI	TWOTRAN II	70
	GMVP	-60
IKE	ICM2D	-196
ANL	VIM	-90
AEC/AEK	RED-1	4588
	RED-2	4111
<i>IFE</i>	<i>FCM2D</i>	<i>43</i>
	<i>VNEM2D</i>	<i>-39</i>

# verification plan for actual plants



# Ringhals-3, case 1



**B bank : 228steps**

**C bank : 228steps**

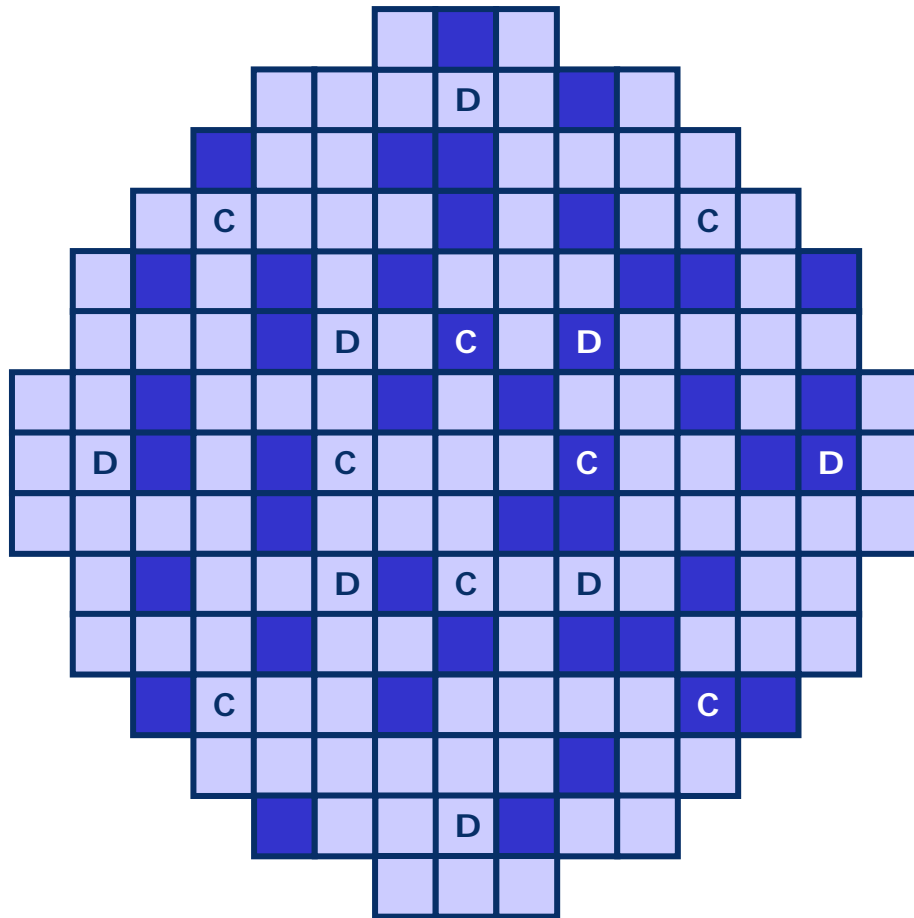
**D bank : 228steps**

**boron : 1315ppm**

**power : 4%**

 : **measured assembly**

## Ringhals-3, case 2



**B bank : 228steps**

**C bank : 111steps**

**D bank : 0steps**

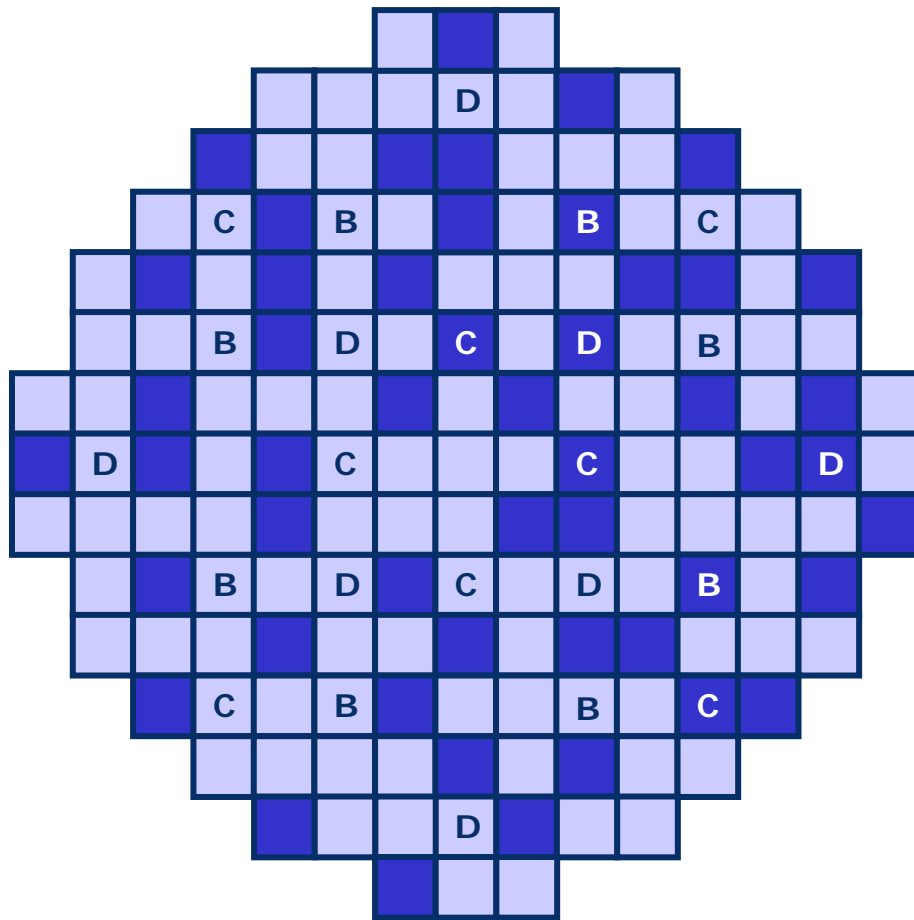
**boron : 1131ppm**

**power : 4%**

 : measured assembly



# Ringhals-3, case 3



**B bank : 217 steps**

**C bank : 0 steps**

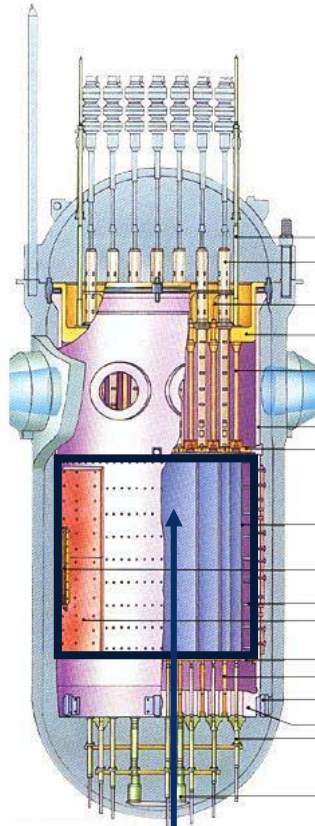
**D bank : 0 steps**

**boron : 1060 ppm**

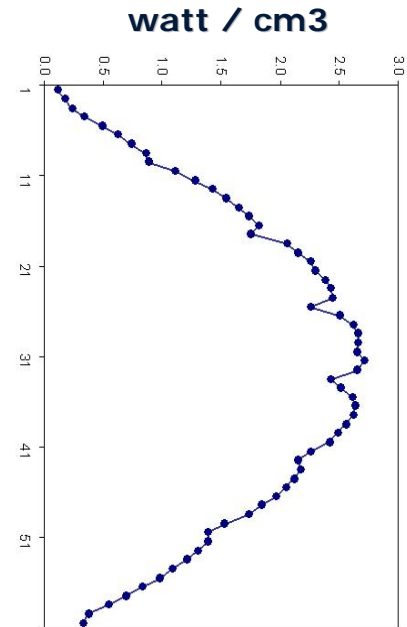
**power : 4%**

 : **measured assembly**

# measured data from plant

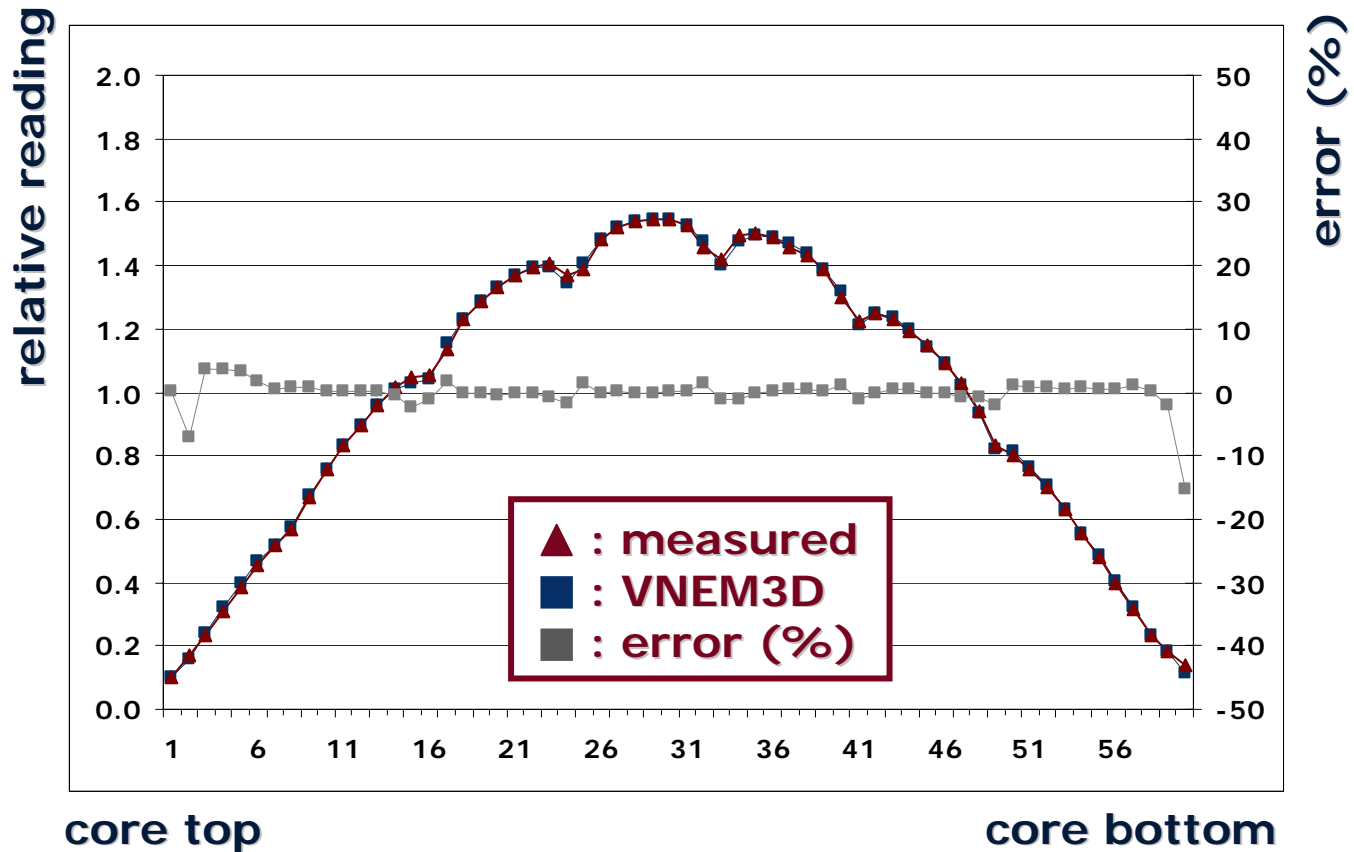


movable detector

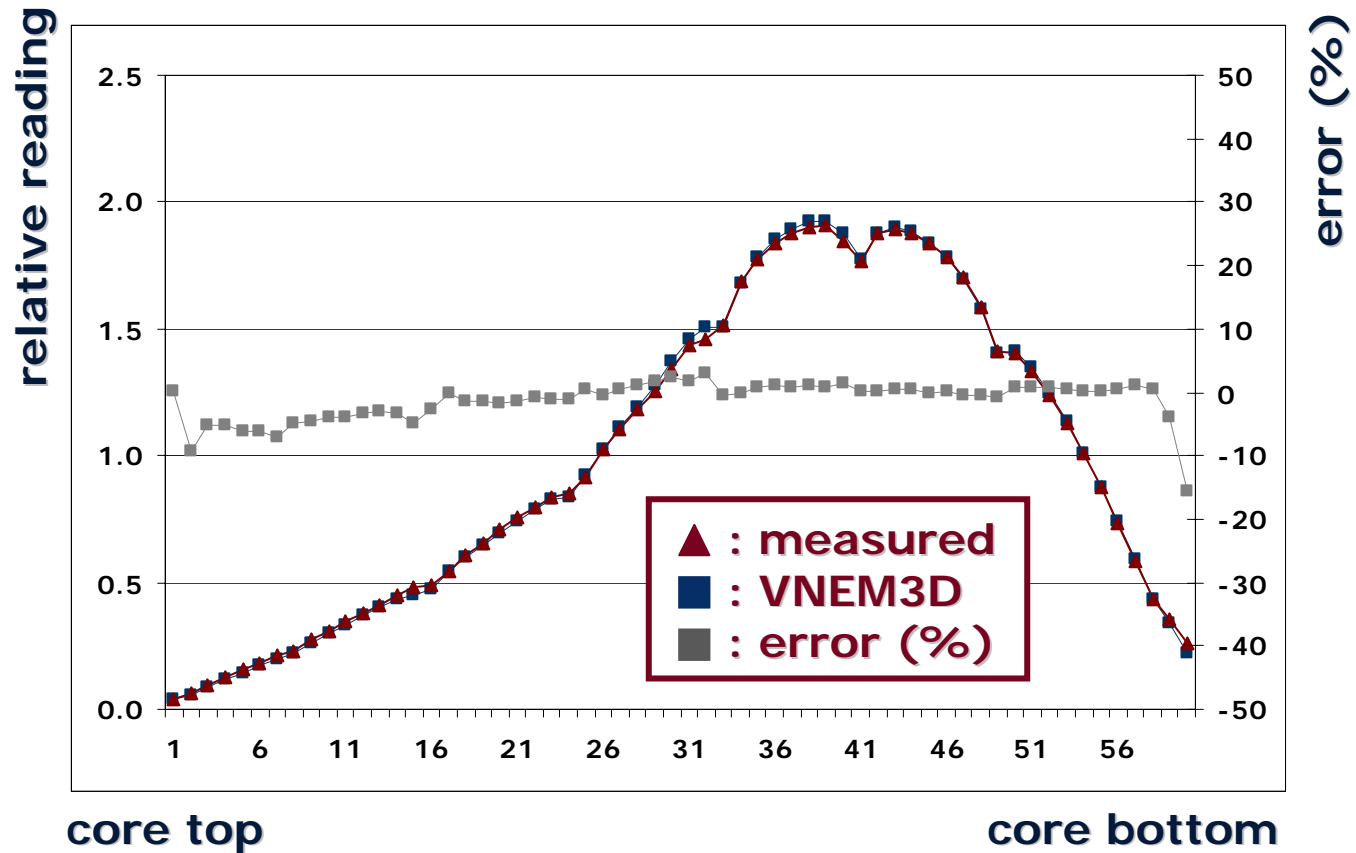


detector readings

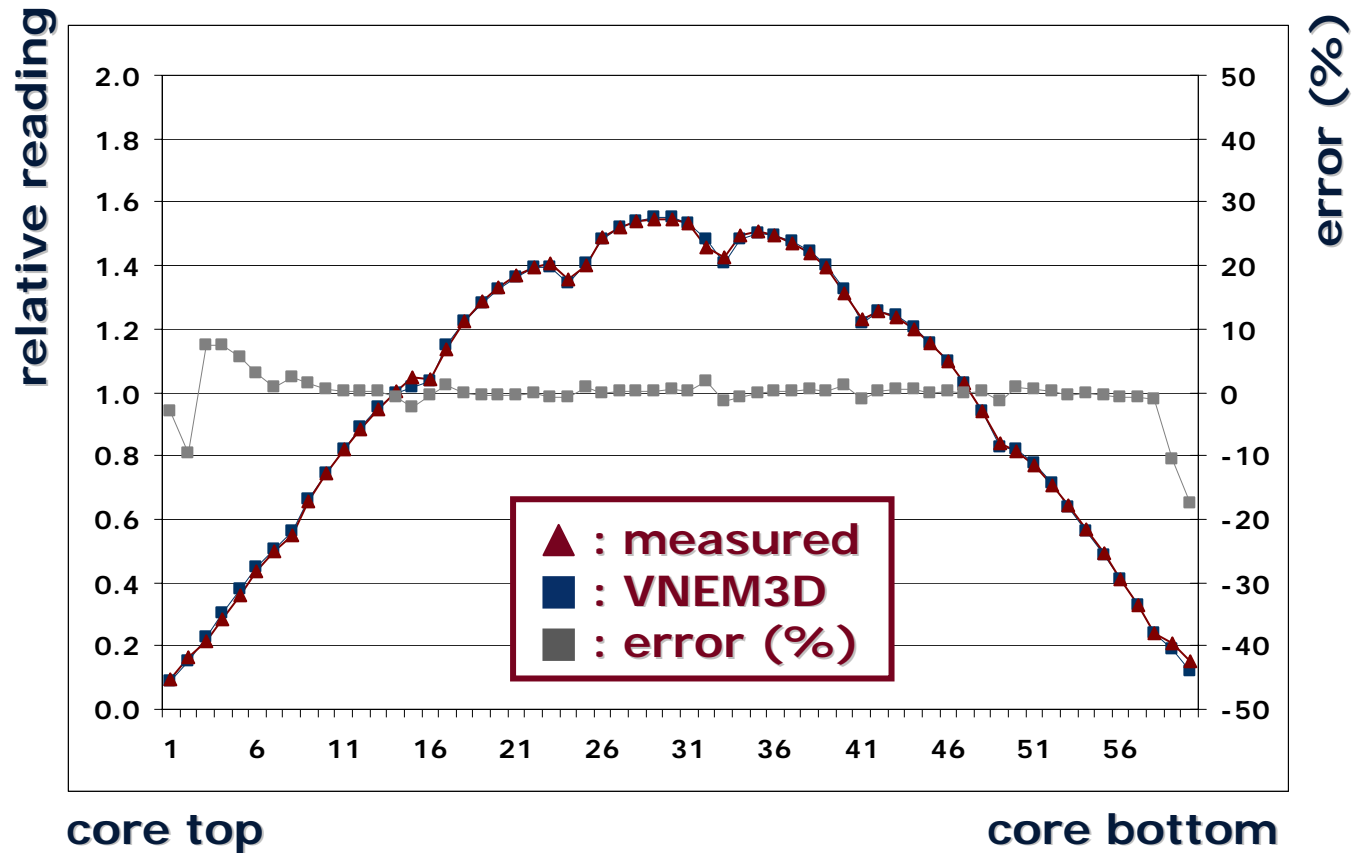
# core average axial detector readings (average = 1.0), case 1



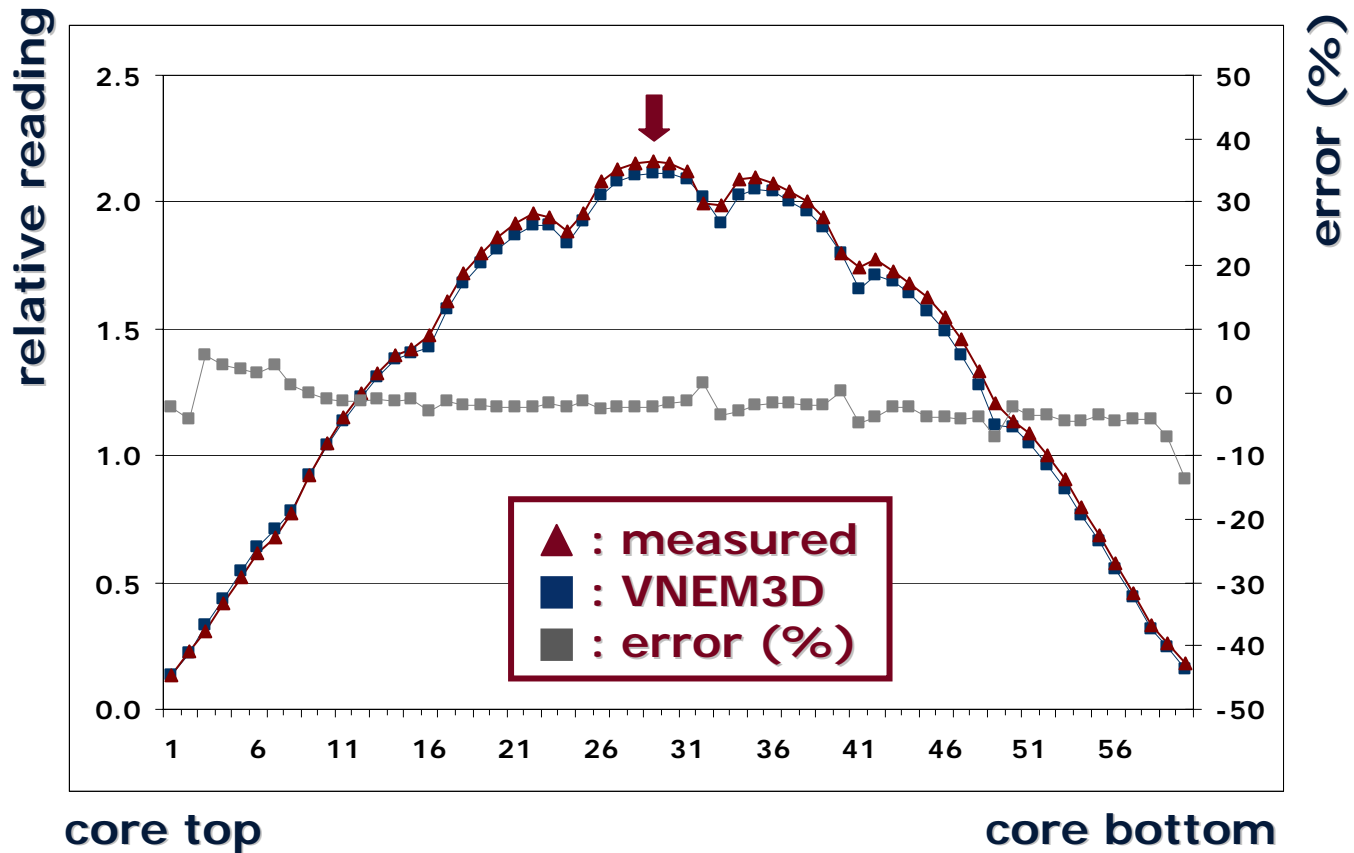
# core average axial detector readings (average = 1.0), case 2



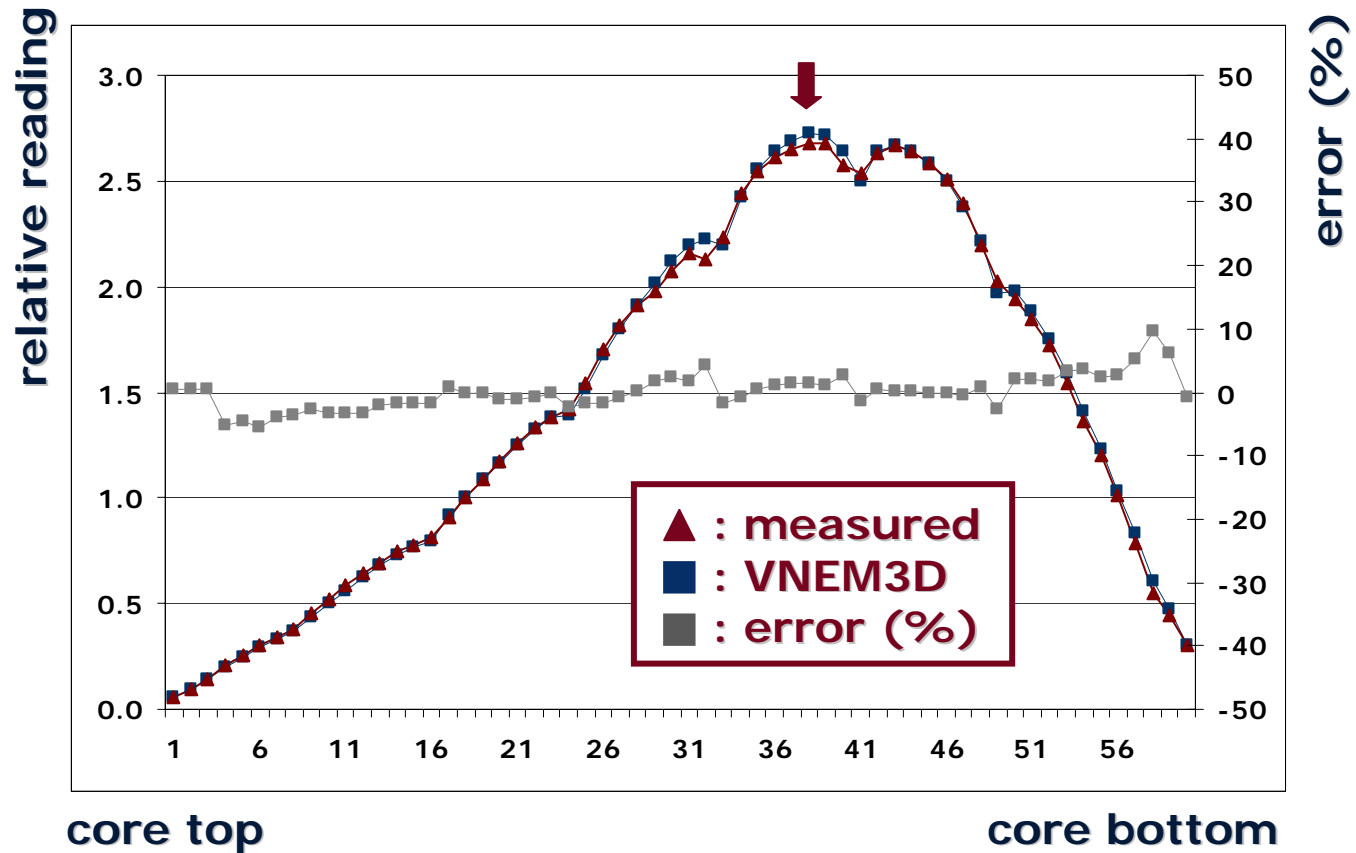
# core average axial detector readings (average = 1.0), case 3



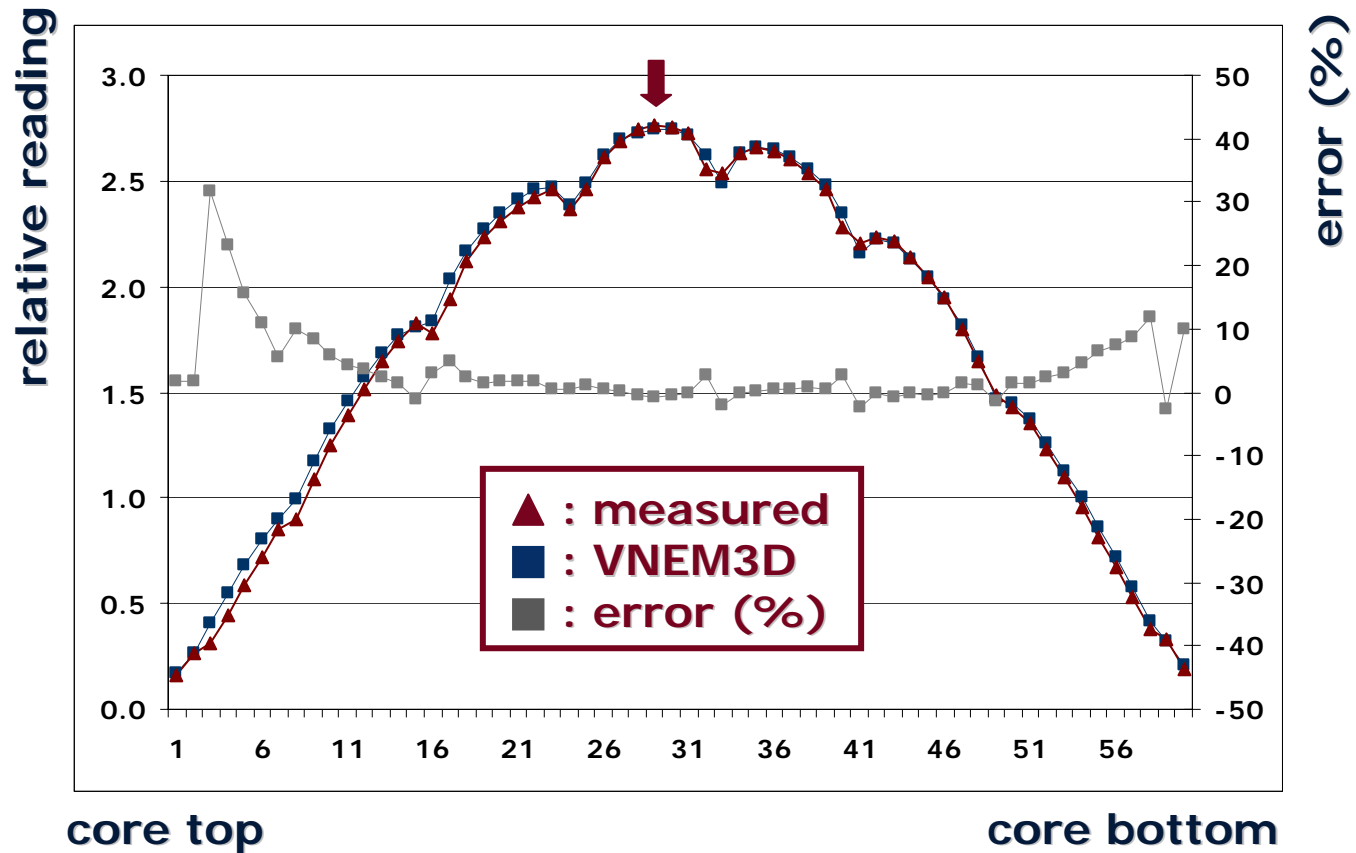
# core maximum detector reading (relative to core average), case 1



# core maximum detector reading (relative to core average), case 2



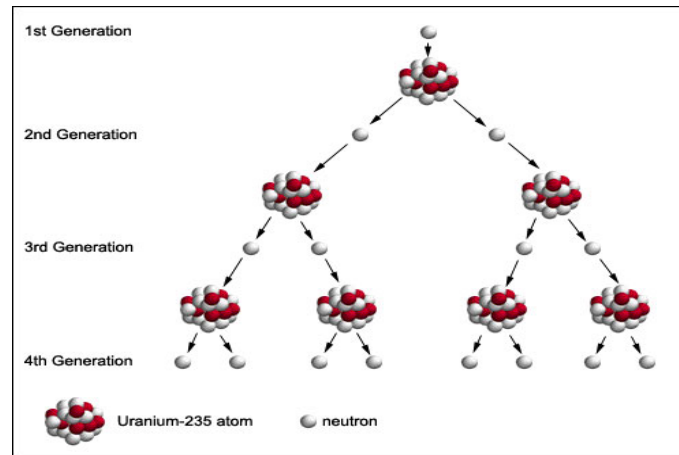
# core maximum detector reading (relative to core average), case 3





# neutron multiplication factor

case	keff	error(%)
1	0.9983	-0.17
2	0.9998	-0.02
3	0.9996	-0.04



# conclusion

1. excellent agreement of power distribution and effective multiplication factor
2. transport effect (P1:P3) is significant
3. large sensitivity on number of groups
4. computing time = 10min./case



# cooperation

- **Rinhgals NPP - Sweden**
- **SEPCO - Japan**
- **TEPSYS (TEPCO) - Japan**
  
- **NTNU - Norway (master student)**
- **VTT - Finland**

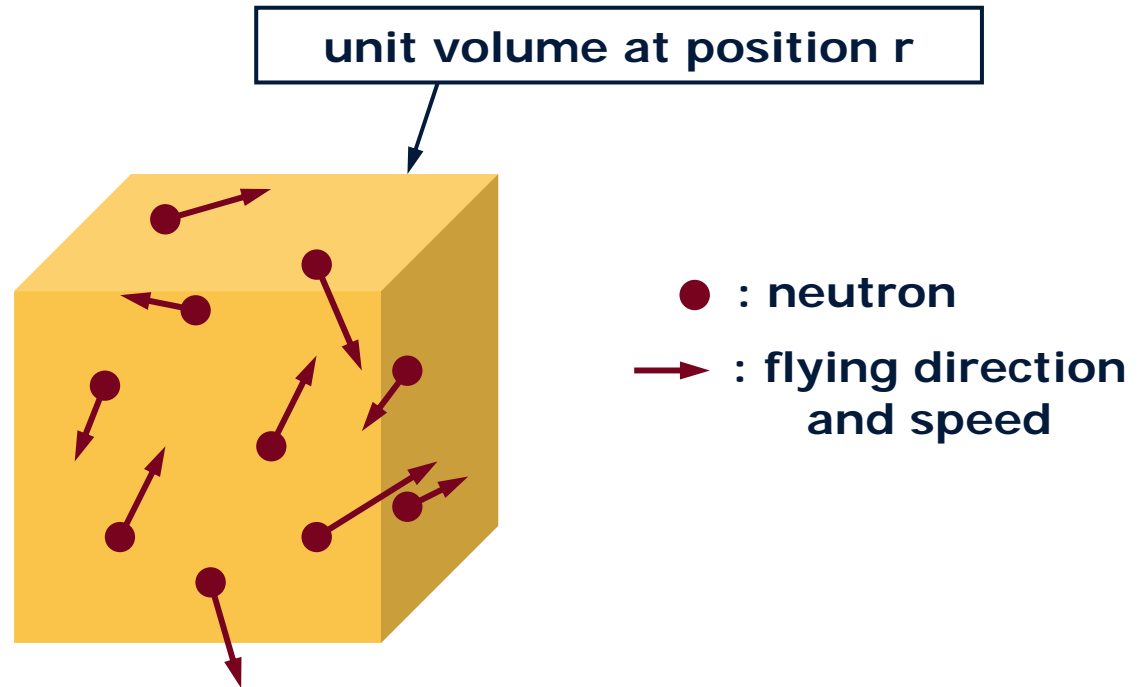


## work in 2009, 2010

- 1. Develop VNEM coefficient generator: VCOEF code**
- 2. Develop VNEM coefficients tabulator: VTABLE code**
- 3. Burnup-tilt model development**
- 4. Make VNEM module faster**
- 5. Built in VNEM module into CYGNUS code**
- 6. verification**



# conservation law of neutrons flying in unit volume in core



count number of neutrons in unit volume at position  $r$ ,  
speed (energy)  $g$ , direction  $\underline{\Omega}$

# energy group condensation

energy-condensed angular flux:

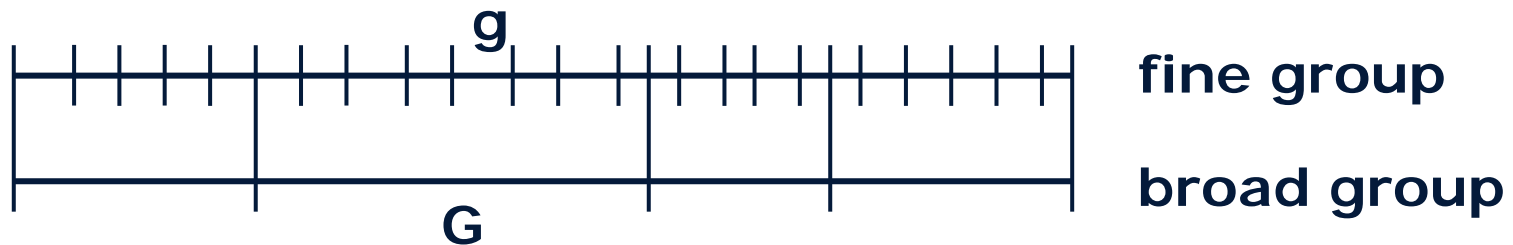
$$\Phi_G(\mathbf{r}, \Omega) \equiv \sum_{g \in G} \Phi_g(\mathbf{r}, \Omega) \Delta E_g$$

energy-condensed neutron cross section:

$$\Sigma_G(\mathbf{r}) \equiv \sum_{g \in G} \Sigma_g(\mathbf{r}) \Phi_g(\mathbf{r}) \Delta E_g / \Phi_G(\mathbf{r})$$

scalar neutron flux:

$$\Phi_g(\mathbf{r}) \equiv \int_{4\pi} d\Omega \Phi_g(\mathbf{r}, \Omega)$$



# homogenization (VNEM)

PL method for angular dependence:

$$\Phi_G(\mathbf{r}, \Omega) \equiv \sum_{l, m} \phi_{G, l, m}(\mathbf{r}) Y_{l, m}(\Omega) \quad \leftarrow \text{spherical harmonics}$$

spatial expansion of moments:

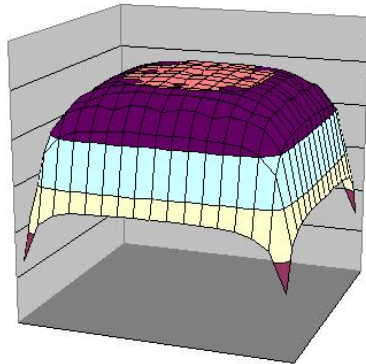
$$\begin{aligned} \phi_{G, l, m}(\mathbf{r}) = & \sum_{nr} F_{G, nr}^S \Psi_{G, nr, l, m}^S(\mathbf{r}) \\ & + \sum_{pp, lmcq} F_{G, pp, lmcq}^{CB} \Psi_{G, pp, lmcq, l, m}^{CB}(\mathbf{r}) \\ & + \sum_{pp, lmsq} F_{G, pp, lmsq}^{SB} \Psi_{G, pp, lmsq, l, m}^{SB}(\mathbf{r}) \end{aligned}$$

calculate in stage 1

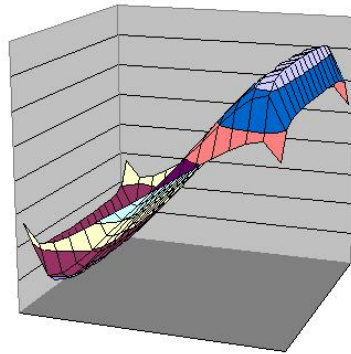
reduce the size of the problem (Ritz method):

**solve  $\Phi_g(\mathbf{r}, \Omega)$  to solve  $F_{G, nr}^S, F_{G, pp, lmcq}^{CB}, F_{G, pp, lmsq}^{SB}$**

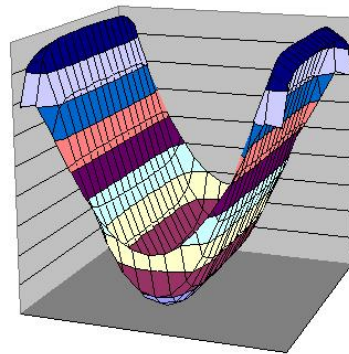
# flux moment nodal expansion



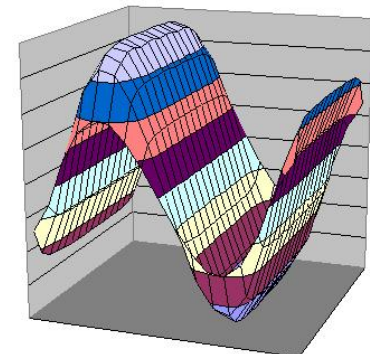
$S_{n=0,g=1}(x,y)$



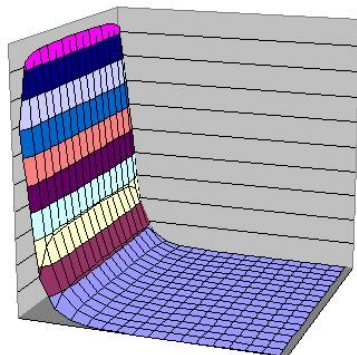
$S_{n=1,g=1}(x,y)$



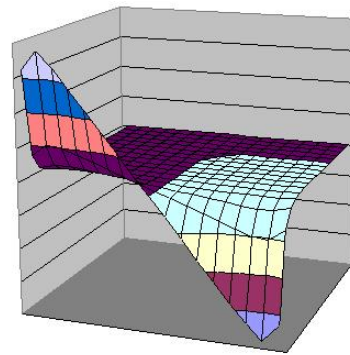
$S_{n=2,g=1}(x,y)$



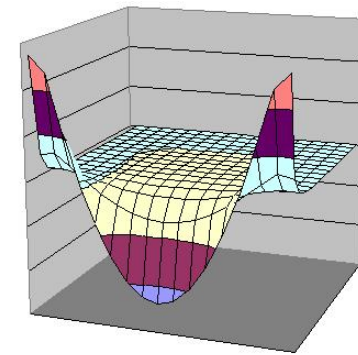
$S_{n=3,g=1}(x,y)$



$B_{m=0,g=1}(x,y)$



$B_{m=1,g=1}(x,y)$



$B_{m=2,g=1}(x,y)$



# number of energy groups, PL-order

## Case 2

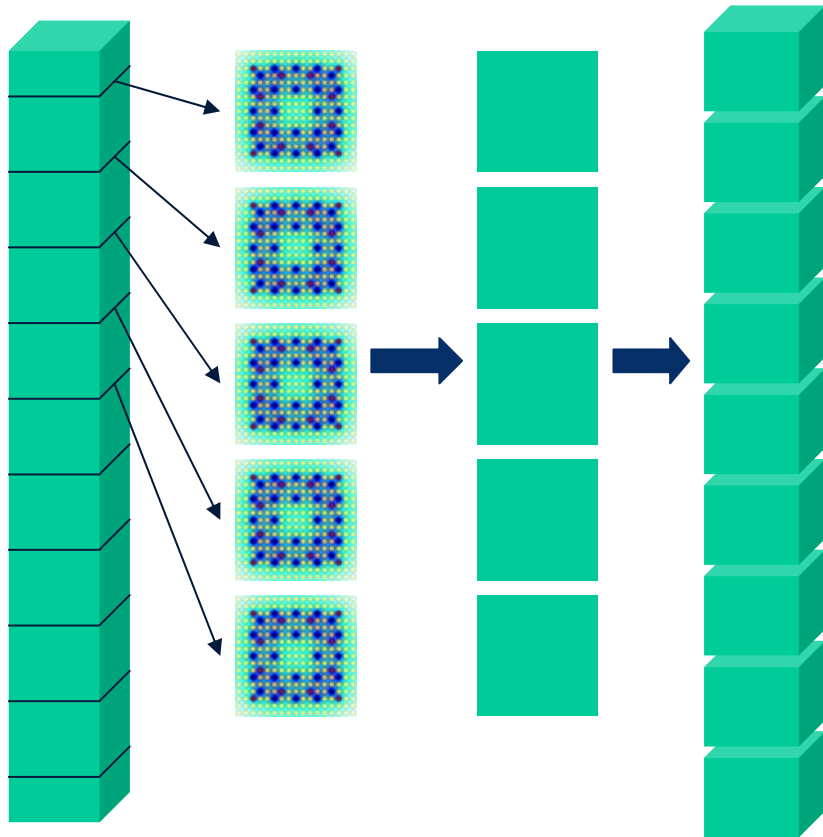
	dkeff (%)	rms (%) overall	axial	radial
7G	reference	3.1	1.5	2.0
5G	0.04	3.1	1.5	2.0
4GA	0.17	4.6	1.6	3.4
4GB	0.46	9.9	2.6	7.7
3G	1.02	21.8	3.3	17.8
(7GP1	0.04	4.6	1.6	3.5)

## Case 1

	dkeff (%)	rms (%) overall	axial	radial
7G	reference	3.4	2.7	1.4
5G	-0.01	3.5	2.8	1.5
(7GP1	-0.01	4.1	2.7	2.6)



# method – stage 1 of 2



2D cross sectional  
calculation, infinite-  
lattice assumption

"homogenized"  
node and  
assembly

- solve **2D** transport equations
- detailed geometry
- with sufficiently large no. of energy groups
- **assuming reflective boundary condition**



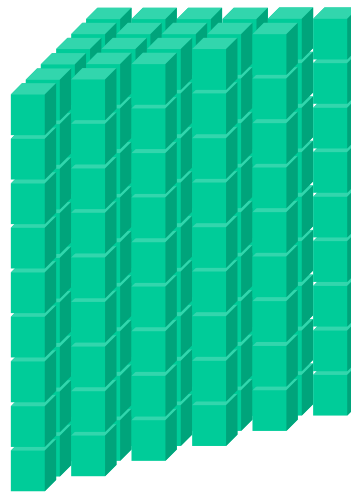
- **homogenization of cross sectional area**
- **energy group condensation**

# method – stage 2 of 2

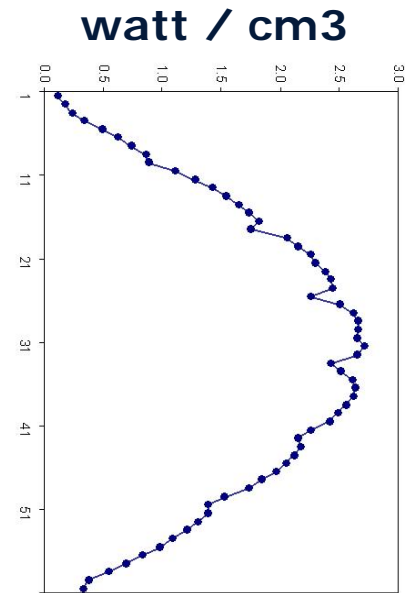


"homogenized"  
node and  
assembly

- results of stage 1 are tabulated
- solve **3D** (homogenized) nodal transport equations
- including feedback effects



"nodal" core  
model



power  
distribution,  
keff