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CCF MODEL COMPARISON

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Abstract

The report describes a simple comparison of two CCF-models, the ECLM, and the Beta-model. The objective of the comparison is to identify differences in the results of the models by applying the models in some simple test data cases. The comparison focuses mainly on theoretical aspects of the above mentioned CCF-models. The properties of the model parameter estimates in the data cases is also discussed. The practical aspects in using and estimating CCF-models in real PSA context (e.g. the data interpretation, properties of computer tools, the model documentation) are not discussed in the report. Similarly, the qualitative CCF-analyses needed in using the models are not discussed in the report.

Key words

Comparison, CCF-models, ECLM, Beta-model

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CCF MODEL COMPARISON

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1 Introduction

This report describes a simple comparison of two CCF-models, ECLM, (see Mankamo (2001a)) and Beta-model (see Alm (2001b)&(2001c)). The comparison was initiated by the Nordic PSA group (NPSAG) in 2002, due to the findings by Alm (2001a) about the Mankamo's Extented Common Load Model and Mankamo's response (Mankamo 2001b, 2002a, 2002b) on Alm's comments.

The objective of the comparison is to identify differences in the results of the above mentioned models by applying the models in some simple test data cases. The models were applied in these cases by Mankamo (2002c) and Alm & Parviainen (2002). The comparison presented here focuses mainly on theoretical aspects of the above mentioned CCF-models. The properties of the model parameter estimates in the data cases is also discussed. The practical aspects in using and estimating CCF-models in real PSA context (e.g. the data interpretation, properties of computer tools, the model documentation) are not discussed in this report. Similarly, the qualitative CCF-analyses needed in using the models are not discussed in this report.

In this report, the models and their estimation principles are first shortly reviewed (section 2), and the results of comparisons are described in section 3. Finally, some conclusions are given.

2 Models and their estimation

The models to be compared in this report are both based on the Common Load Model (CLM), (Mankamo 1977). The basic assumption of CLM is that redundant components are operating under common load or stress, which is a random variable. The strengths of the components are independent and identically distributed random variables, and a failure occurs when the load exceeds the strength. The probability that k out of n components fail is according to the CLM

$$P(k \text{ components out of } n \text{ fail}) = \binom{k}{n} \int_x F_R(x)^k (1 - F_R(x))^{n-k} f_L(x) dx, \qquad (1)$$

where $f_L(x)$ is the probability density function of the common load and $F_R(x)$ is the cumulative distribution of the strength (resistance) of the components. In the original CLM, the distributions of the common load and component resistances are Gaussian.

Because there is some evidence that the multiple failures of highly redundant systems may have stronger dependence, Mankamo (1995, 2001) introduced an new version of CLM, i.e. the Extented Common Load Model (ECLM), in which the load variable is described by a two component Gaussian distribution,

$$X_L \sim w_b N(x_{nb}, \sigma_b) + w_x N(x_{nx}, \sigma_x), \qquad (2)$$

where w_b is the mixing parameter, i.e. the proportion of basic load part ($w_x = 1 - w_b$ is the proportion of extreme loiad part). The extreme load part corresponds to the very high stresses (~ dependence) which may caus CCFs with very high multiplicity. Mankamo (xxx) uses a paramnetrisation of ECLM, in which the density function of the strength distribution is

$$f_R(x) = \varphi_{1,\sigma_R}(x) = N(1, \sigma_R), \tag{3}$$

where $\varphi_{1,\sigma_R}(x)$ is the gaussian denszity with expected value *1* and standard deviation σ_R. The parametrisation of basic load part of the load distribution is

$$f_{Lb}(x) = \varphi_{o,\sigma_b}(x) = N(0, \sigma_b), \tag{4}$$

and the parametrisation of the extreme load part is

$$f_{Lx}(x) = \varphi_{o,\sigma_x}(x) = N(y_x, \sigma_x). \tag{5}$$

In this report, the predictions of ECLM are compared with those of so called Beta model, developed by Alm (200x). The Beta model is based on the observation that the strength variable can be transformed to the interval [*0,1*] by defining a new variable

$$X_R^{'} = F_R(X_R), \tag{6}$$

where $F_R(x)$ is the cumulative distribution of the strength. The transformed strength variable follows uniform *U(0,1)*-distribution, and the equation (1) has the form

$$P(k \text{ components out of } n \text{ fail}) = \binom{k}{n} \int_x x^k (1-x)^{n-k} f_L(x)dx. \tag{7}$$

In the Beta model, it is assumed that the load variable follows a beta distribution

$$X_L \sim \text{Beta}(\alpha, \beta). \tag{8}$$

It is possible to transform ECLM to the interval [*0,1*] as in equations (7)-(8). The form of equation (7) reveals that all CLM models can be interpreted as kind of distributed failure probability models.

The estimation of the models is based on maximum likelihood principle (see Mankamo 2001 and Alm 2001b). The data used in estimation consists of numbers of observed failures of different multiplicity, and the likelihood is essentially a product of probabilities of the form (1). The maximum likelihood estimates are found by a numerical procedure. In principle, the confidence bounds of the parameter estimates could be determined by using the Fishers information matrix (i.e. the expected values of second partial derivatives of the log-likelihood). However, they have not been calculated for the models. Instead of this, Mankamo (2001) has made some experiments with Bayesian estimation.

3 Comparison of the models

3.1 Comparison principles

The models are compared by estimating their parameters on the basis of 5 data sets. The multiple failure probabilities are then compared. In addition to this, the estimated loads distributions are compared. In order to do that, the domain of ECLM is transformed onto the unit interval according to the transformation (9). The density function of the transformed (ECLM) load distribution is

$$f_L(x) = (\varphi_{1,\sigma_R}(\Phi^{-1}_{1,\sigma_R}(x))^{-1} \times \left\{w_b\varphi_{0,\sigma_b}((\Phi^{-1}_{1,\sigma_R}(x)) + w_x\varphi_{y_x,\sigma_x}((\Phi^{-1}_{1,\sigma_R}(x))\right\}, x \in (0,1) \qquad (9)$$

If ECLM is transformed according to (9), the strength distribution is the uniform *U(0,1)*-distribution, as in the case of the Beta-model..

Similarly, it is possible to transform the Beta model in such way that it's domain is *(-∞,∞)* and it's strength distribution is identical to that of ECLM. The density function of the load of the transformed Beta model becomes

$$f_R(x) = \frac{\varphi_{1,\sigma_R}(x)}{B(\alpha,\beta)}\left[\Phi_{1,\sigma_R}(x)\right]^{\alpha-1}\left[1-\Phi_{1,\sigma_R}(x)\right]^{\beta-1}, x \in (-\infty,\infty)\,.$$
(10)

It is also possible to compare the goodness of fit using the maximal likelihood values. However, in this case the number of parameters of ECLM is larger than that of Beta model, and it is clear that it has "better" likelihood values. One possibility to make a likelihood-based evaluation of the goodness is to use some kind of Akaike criterion.

3.2 Data set 1

The first data set is based on the OL1/OL2 experience and it is suggested by Mankamo. It consists of five different variants which are formed for purpose of sensitivity analysis, The data set is in Table 1. The fractional numbers of failures (e.g. 0.8 failures of multiplicity 3 in the data set 1.1) are based on the impact vector assumption, in which some failure events are interpreted uncertain.

Table 1. Data set 1

Multiplicity	Data set 1.1	Data set 1.2	Data set 1.3	Data set 1.4	Data set 1.5
0	26.5	27	26	26	26
1	5	5	5	5	5
2	1	0	2	2	2
3	0.8	0.8	1	1	1
4	0.5	1	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	0.15	0	0	1	2
8	0	0	0	0	0
9	0	0	0	0	0
10	0.05	0.2	0	0	0
Total	34	34	34	35	36

The above data set is rather weak evidence (totally appr. 34 failuree events) for estimating CCF parameters. However, the number of failures with high multiplicity is high, and it should be reflected on the parameter estimates.

The estimated parameter values for the Beta model are in the Table 2. The parameters are rather sensitive to changes in the data. This is understandable due to the small amount of data. In the case of data set 1.3, the estimate of parameter β is large compared to the other cases. This is caused by the fact that in that data set there is no failures of multiplicities 4-10. For data sets 1.4 and 1.5 the ratio of parameters, α/β, is larger. Thus, for these data sets, the Beta-model describes the stronger dependence of failures by the ratio of the parameters. It is worth noticing that the load distribution is modeless in all cases.

Table 2. Parameter estimates of Beta-model for data set 1

Parameter	Data set 1.1	Data set 1.2	Data set 1.3	Data set 1.4	Data set 1.5
α	0.2355	0.16017	0.45372	0.19634	0.16381
β	5.8399	3.6952	12.444	3.3232	2.0597

The parameter estimates for ECLM are in Table 3. The probability of high multiplicity failures is obviously higher in data sets 1.4 and 1.5; in the ECLM parameter estimates this is reflected as larger value of the proportion parameter w_x. The other parameters seem to be rather similar in all data sets, except the data set 1.4. It is interesting to note that in the case of data set 1.2 the variance of the extreme load part is large compared with that of other cases. The probability of high multiplicity failures in data set 1.2 is higher than in data set 1.1; this stronger dependence of failures is now compensated by larger variance of the extreme load distribution. However, the dependence of failures is stronger in data sets 1.4 and 1.5, and it is not possible to describe this by larger variance of extreme load but by larger proportion parameter.

Table 3. Parameter estimates of ECLM for data set 1

Parameter	Data set 1.1	Data set 1.2	Data set 1.3	Data set 1.4	Data set 1.5
w_b	0.98801	0.98712	0.99848	0.87487	0.785
w_x	0.01199	0.01288	0.00152	0.12513	0.215
σ_R	0.434	0.41895	0.46815	0.44932	0.44336
σ_b	0.33977	0.35651	0.29194	0.26633	0.23546
σ_x	0.75171	1.82615	0.57336	0.44932	0.49015
y_x	0.566	0.58105	0.53185	0.55068	0.55664

The comparison of the load distributions is in Figures 1-5. The probability mass of the load distribution of Beta model is generally more concentrated on smaller values of the load variable, which leads to the smaller probabilities of high multiplicity failures.

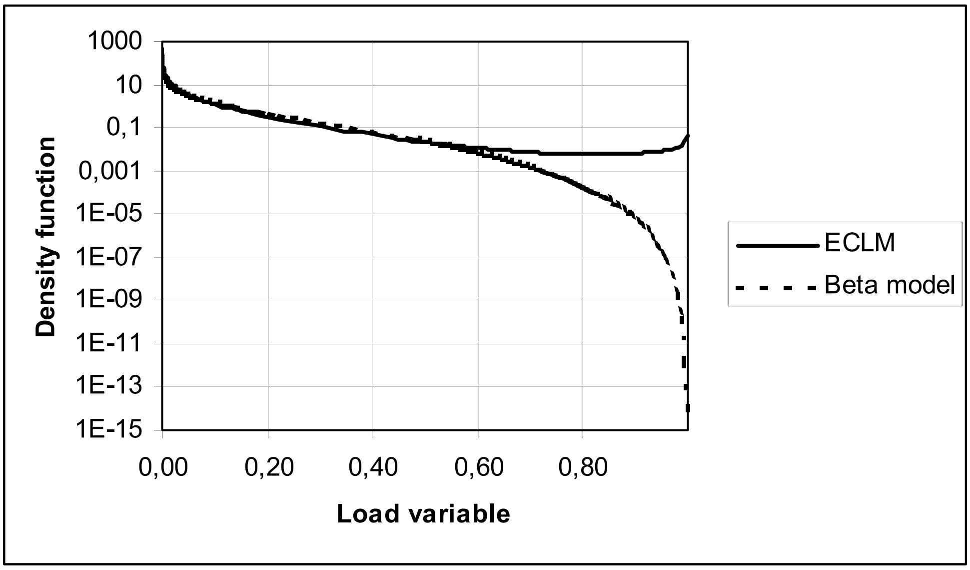

Figure 1 a). Load distributions for data set 1.1. The load distribution of ECLM transformed such that the strength distribution is uniform.

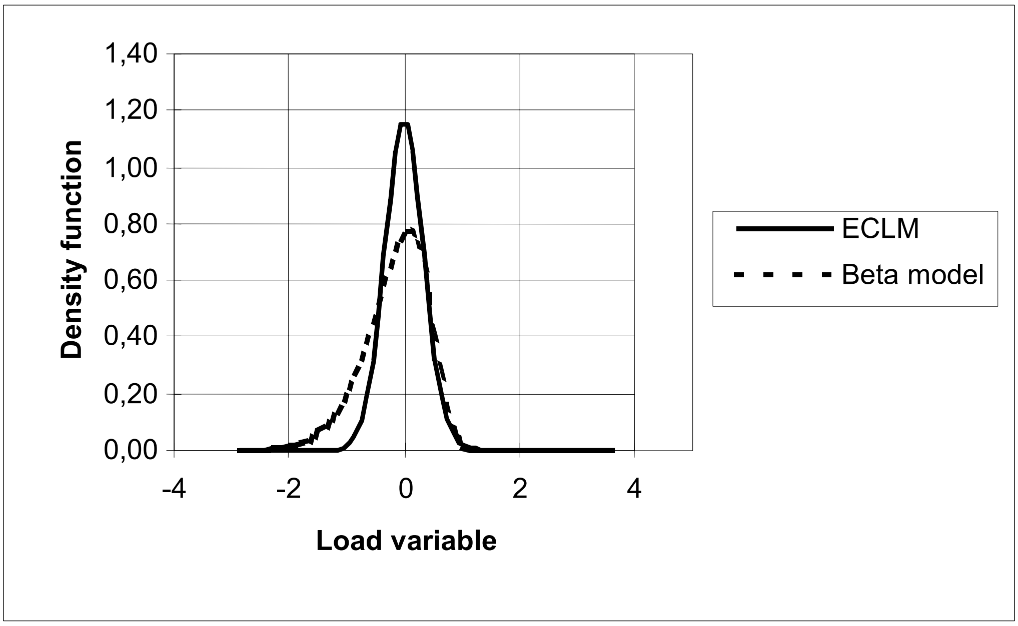

Figure 1 b) Load distributions for data set 1.1. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

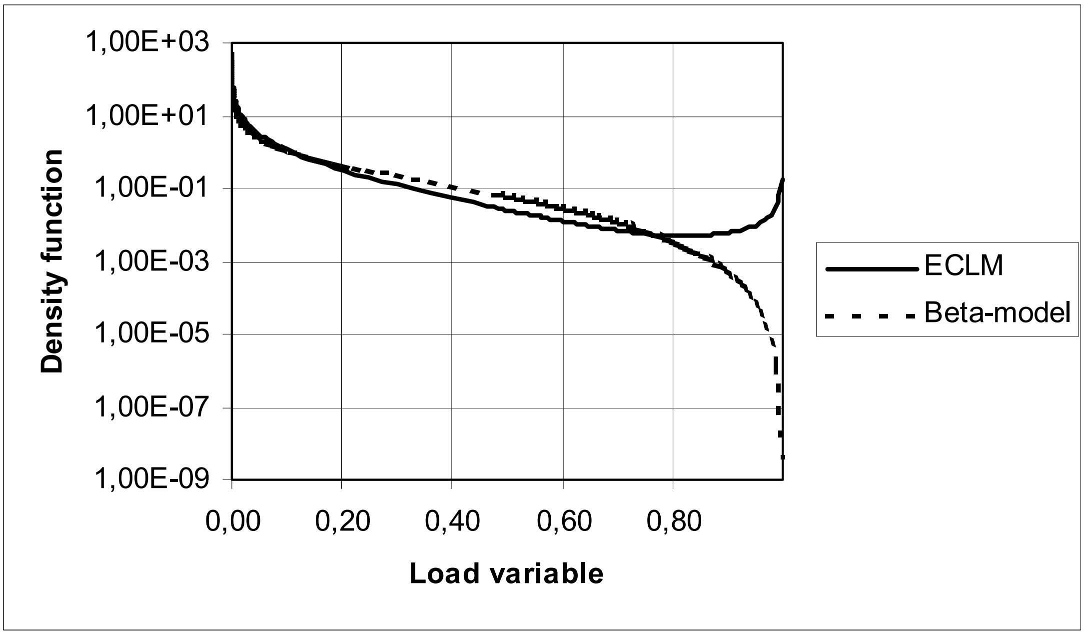

Figure 2 a). Load distributions for data set 1.2. The load distribution of ECLM transformed such that the strength distribution is uniform.

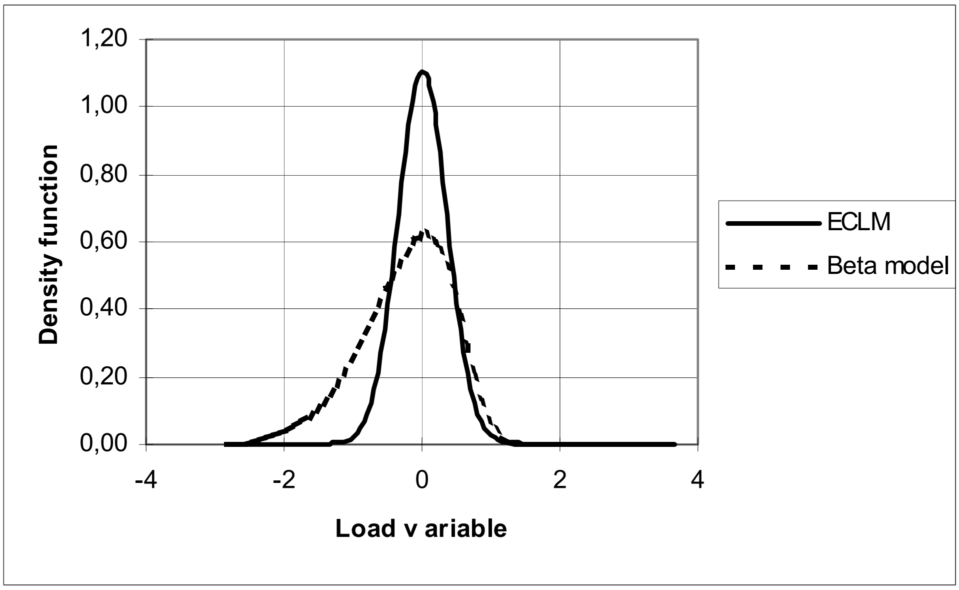

Figure 2 b) Load distributions for data set 1.2. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM ´

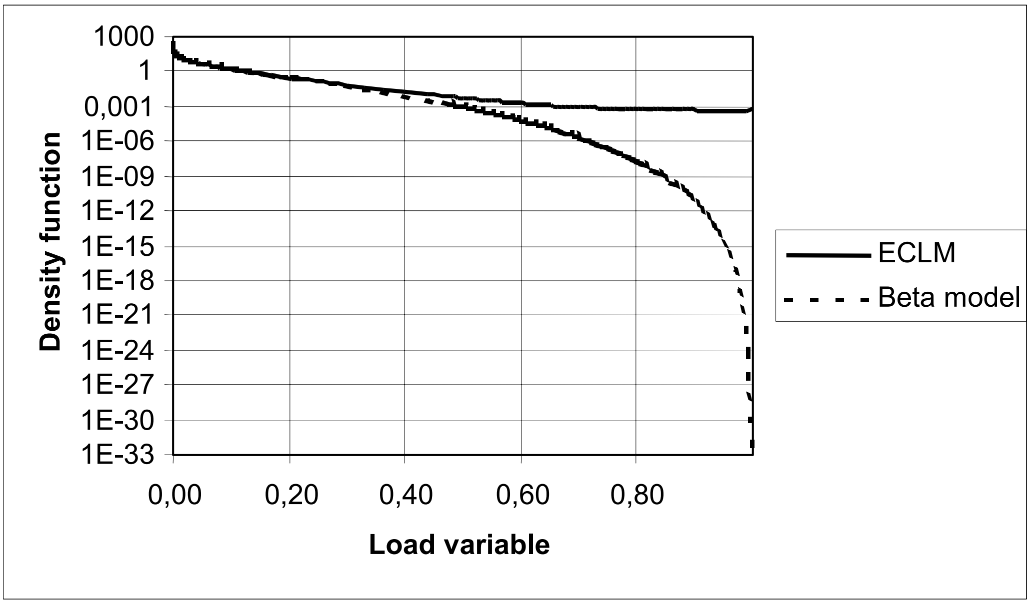

Figure 3a). Load distributions for data set 1.3. The load distribution of ECLM transformed such that the strength distribution is uniform.

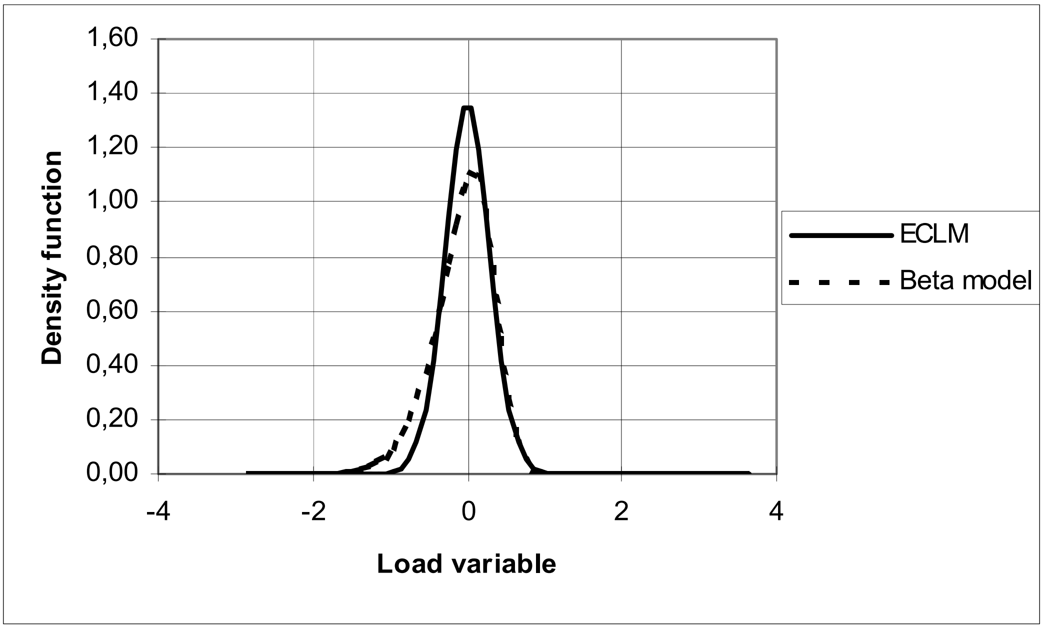

Figure 3 b) Load distributions for data set 1.3. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

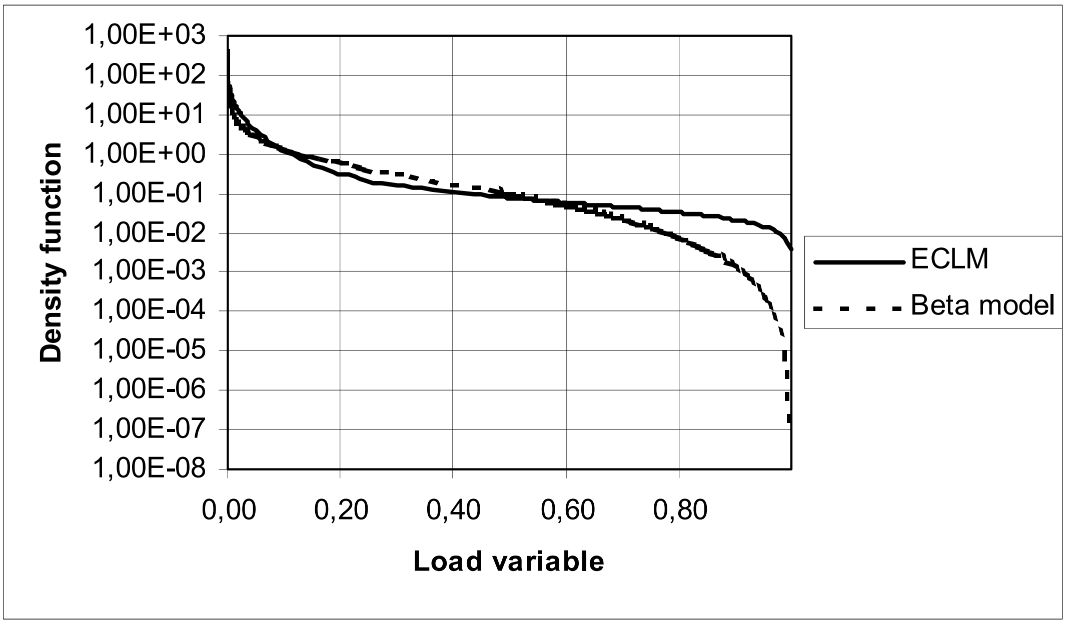

Figure 4a). Load distributions for data set 1.4. The load distribution of ECLM transformed such that the strength distribution is uniform.

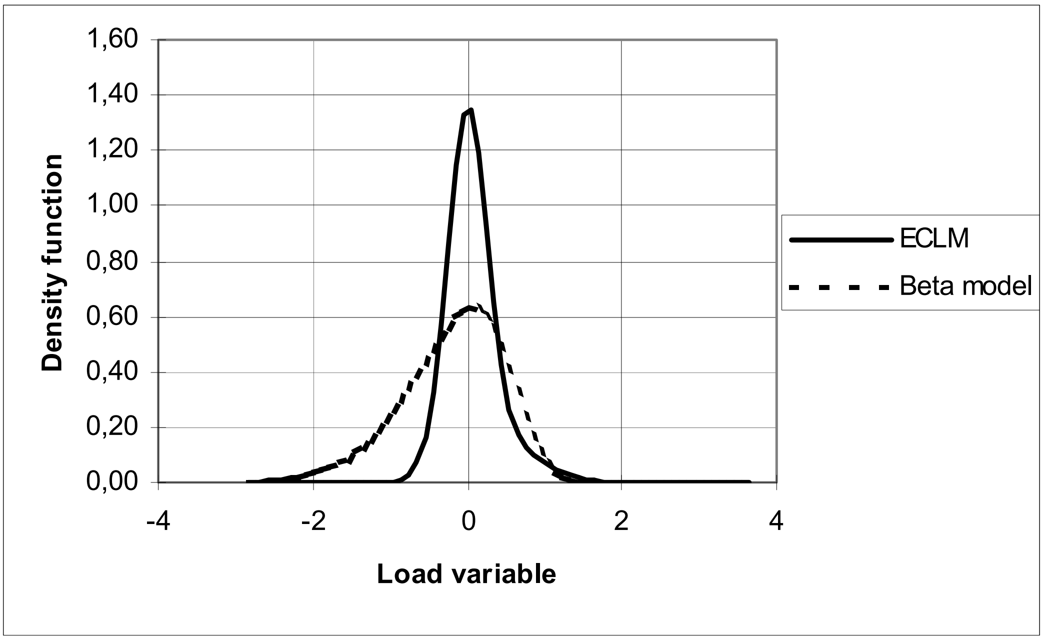

Figure 4 b) Load distributions for data set 1.4. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

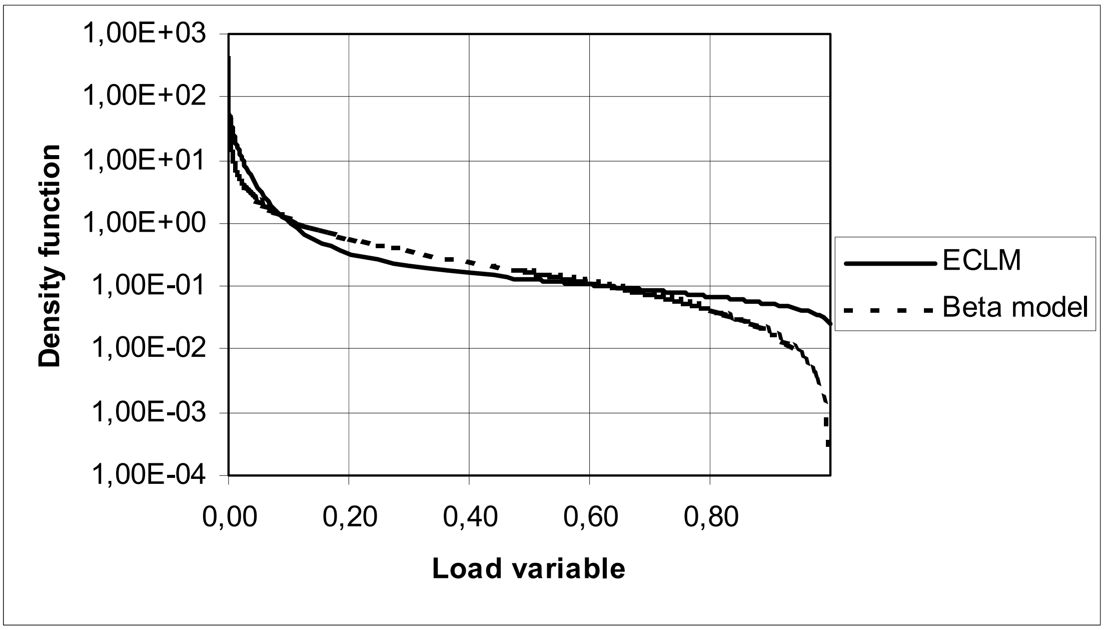

Figure 5a). Load distributions for data set 1.5. The load distribution of ECLM transformed such that the strength distribution is uniform.

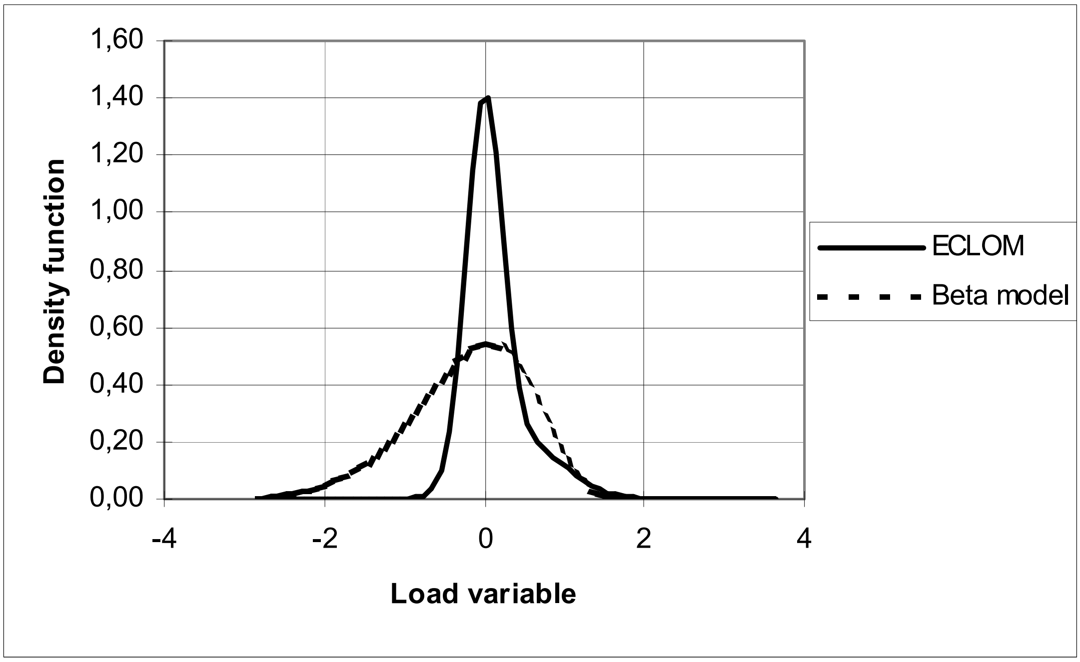

Figure 5 b) Load distributions for data set 1.5. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

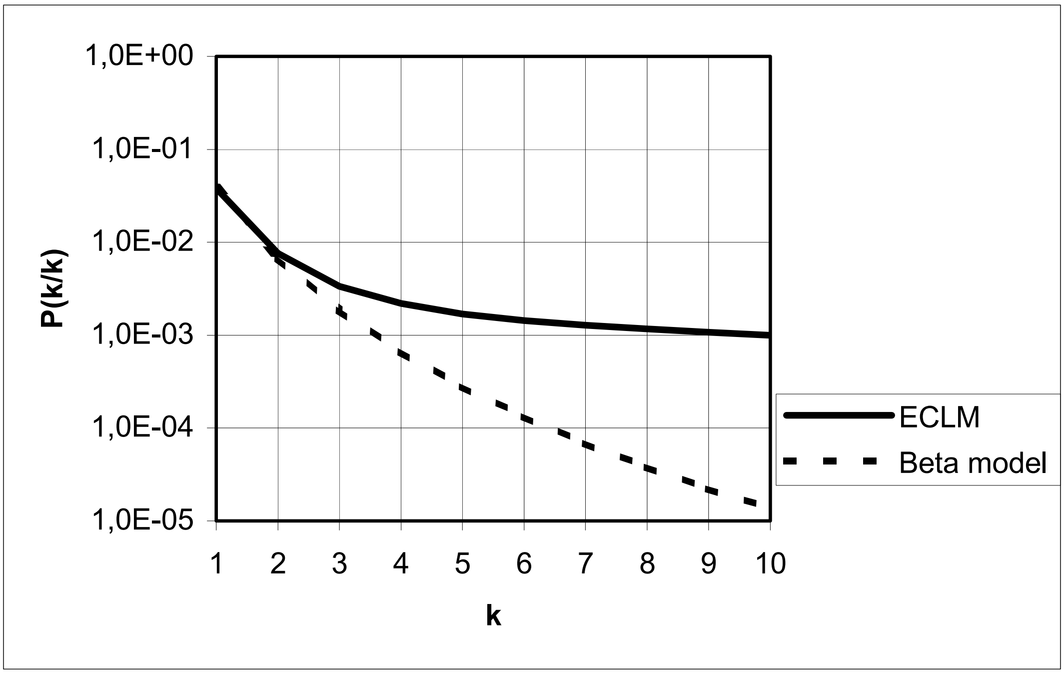

Figure 6. Multiple failure probabilities P(k out of k fail) for data set 1.1
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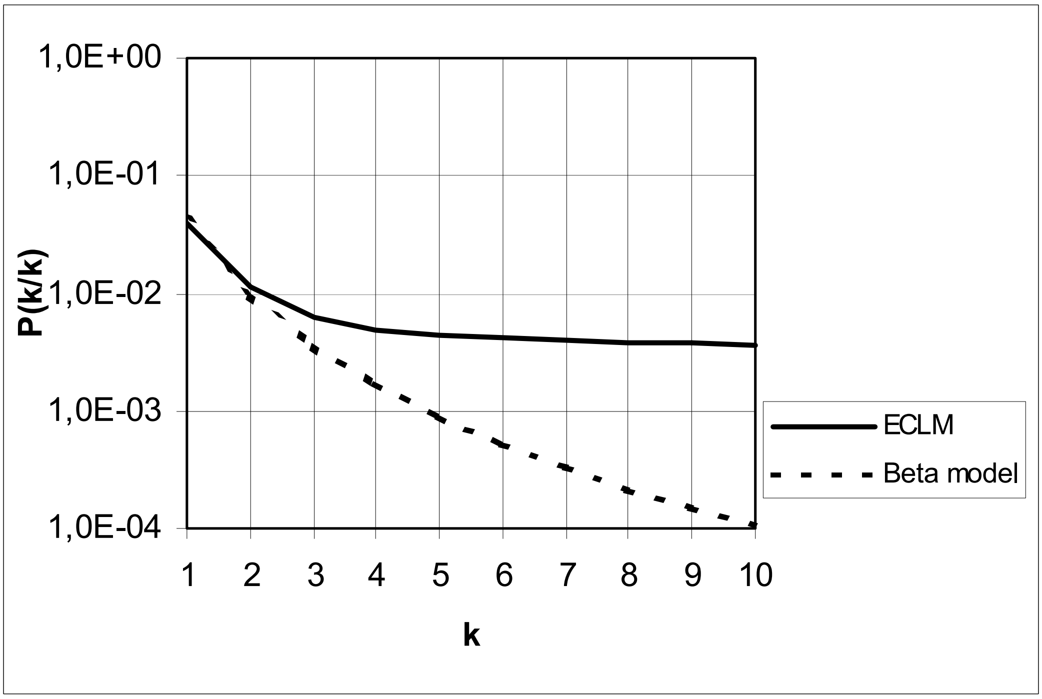

Figure 7. Multiple failure probabilities P(k out of k fail) for data set 1.2.

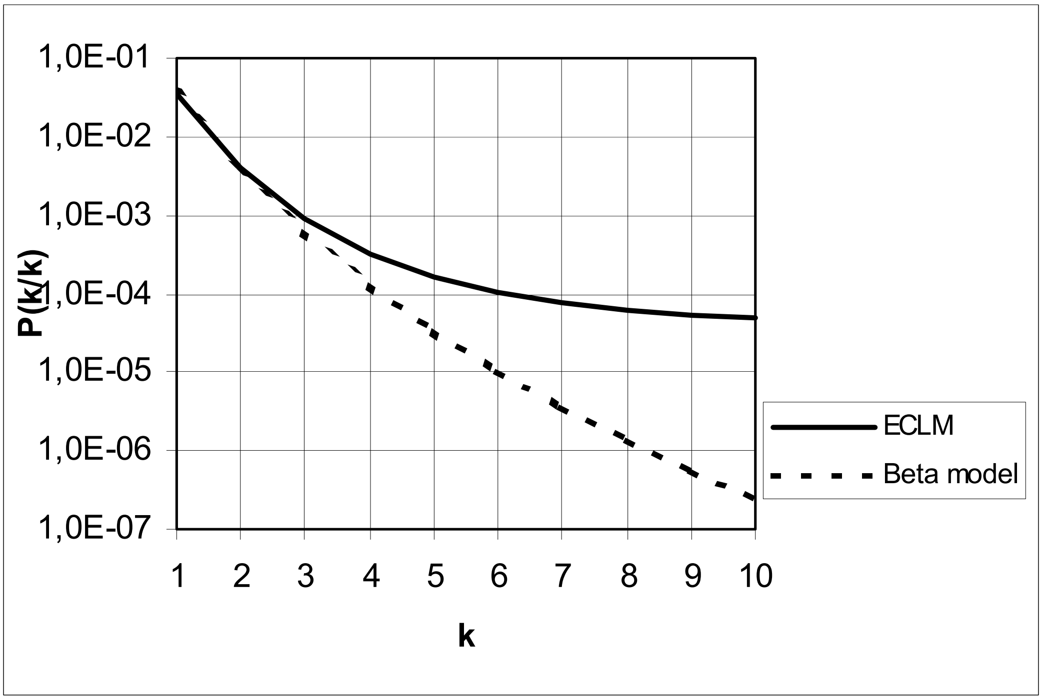

Figure 8. Multiple failure probabilities P(k out of k fail) for data set 1.3

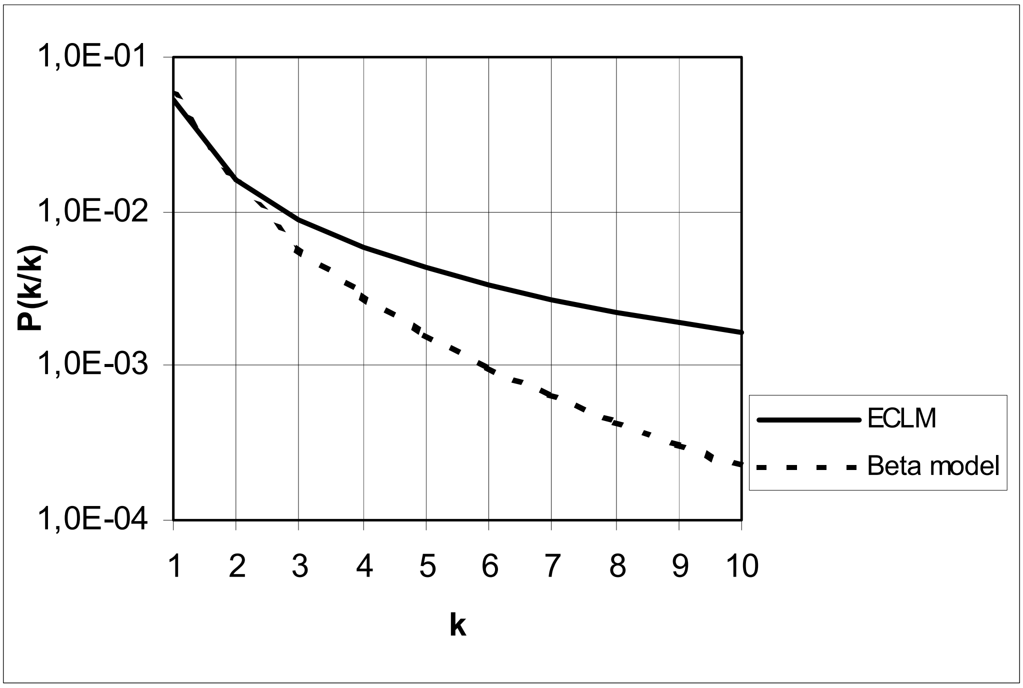

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Figure 9. Multiple failure probabilities P(k out of k fail) for data set 1.4.

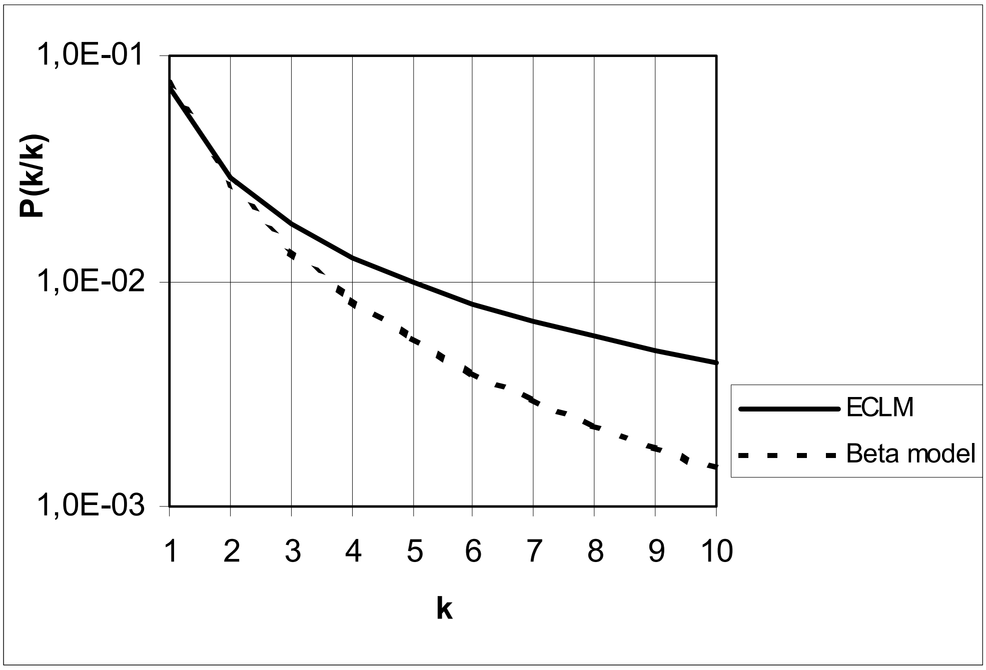

Figure 10. Multiple failure probabilities P(k out of k fail) for data set 1.5.

The probabilities of multiple failures according to the models are presented in Figures 6-10. In all cases, ECLM gives larger failure probabilities *P(k out of k fail)* for large *k*. The single and double failure probabilities do not differ very much. It is important to note that in the cases where it is possible to estimate the probabilities *P(k out of k fail)* directly from the data ECLM gives estimates which are almost exactly the same as those estimated directly from the data.

3.3 Data set 2

The second data set (Table 4) consists of five different variants. The number of system demands is high (~154545). The number of failures of highest multiplicity is relatively large. However, the number of demands with failures with multiplicity 0 or 1 if large. As it can be seen from Table 4, the number of 6-fold failures varies from 4 to 7 in data sets 2.2-2.5, while the number of other failures remains the same.

Table 4. Data set 2.

Multiplicity	Data set 2.1	Data set 2.2	Data set 2.3	Data set 2.4	Data set 2.5
0	15215.2255	15215	15215	15215	15215
1	224.2402	224	224	224	224
2	5,2865	5	5	5	5
3	2,6735	2	2	2	2
4	0,5533	1	1	1	1
5	0,0120	0	0	0	0
6	6	4	5	6	7
Total	155453	15451	15452	15453	15454

The estimated parameter values for the Beta model are in the Table 5 In this case the ratio of parameters is almost the same for all data sets. The parameter values get smaller with increasing number of failures with highest multiplicity. Also in this case the load distribution is modeless in all cases.

Table 5. Parameter estimates of Beta-model for data set 2.

Parameter	Data set 2.1	Data set 2.2	Data set 2.3	Data set 2.4	Data set 2.5
α	0.038404	0.048595	0.042002	0.037033	0.033153
β	12.579	16.622	14.007	12.041	10.511

The parameter estimates for ECLM are in Table 6. F or this data set, the proportion parameter w_x is approximately same for all cases, and the dependence seem to be described by the variance of extreme load part.

Table 6. Parameter estimates of ECLM for data set 2.

Parameter	Data set 2.1	Data set 2.2	Data set 2.3	Data set 2.4	Data set 2.5
w_b	0.99875	0.99885	0.99865	0.99848	0.99836
w_x	0.00125	0.00 115	0.00135	0.00152	0.00164
σ_R	0.33087	0.33218	0.33408	0.33598	0.3341
σ_b	0.1335	0.12841	0.12337	0.11812	0.12337
σ_x	0.99262	0.79074	0.79528	0.7998	0.8643
y_x	0.99913	0.66782	0.66592	0.66402	0.6659

The load distributions are presented for data sets 2.2 and 2.5 (Figures 11 –12). In all cases, the load distribution of Beta model is broader, and it has more probability mass on smaller load values that ECLM. Thus, it is clear that it predicts smaller multiple failure probabilities.

The probabilities of multiple failures according to the models are presented in Figures 13-16. In all cases, ECLM gives larger failure probabilities *P(k out of k fail)* for large *k*. Also in this case, the single and double failure probabilities are rather similar. The Multiple probability 2-6-fold failures of ECLM is almost constant, in Beta model the multiple failure probability decreases rapidly with increasing failure multiplicity. It seems that the failure probability estimates of Beta model are determined mainly by single and double failures.

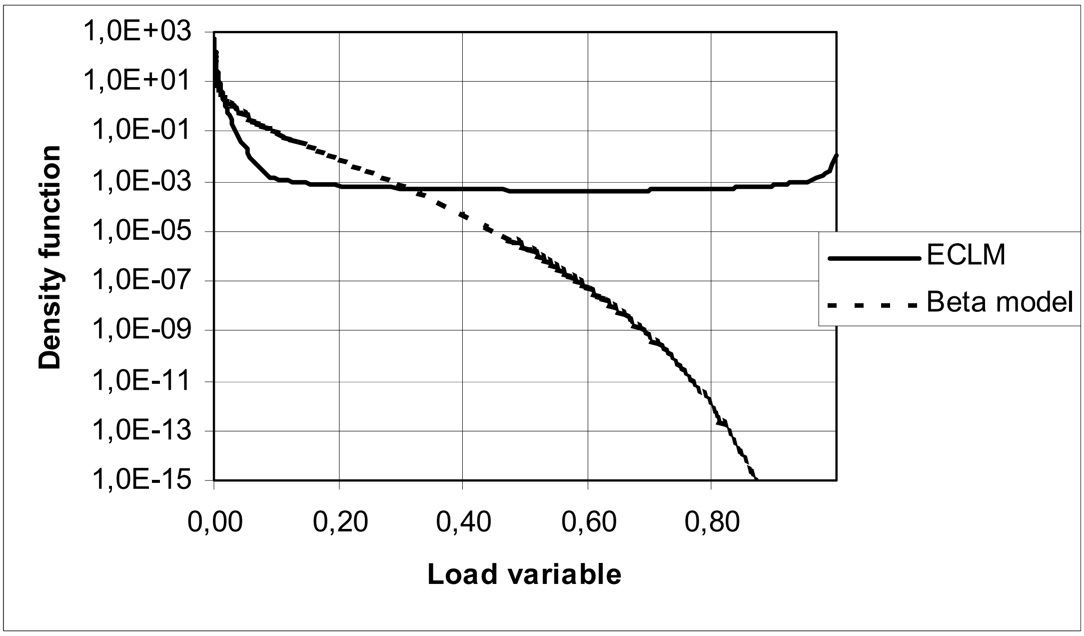

Figure 11 a). Load distributions for data set 2.2. The load distribution of ECLM transformed such that the strength distribution is uniform**.**

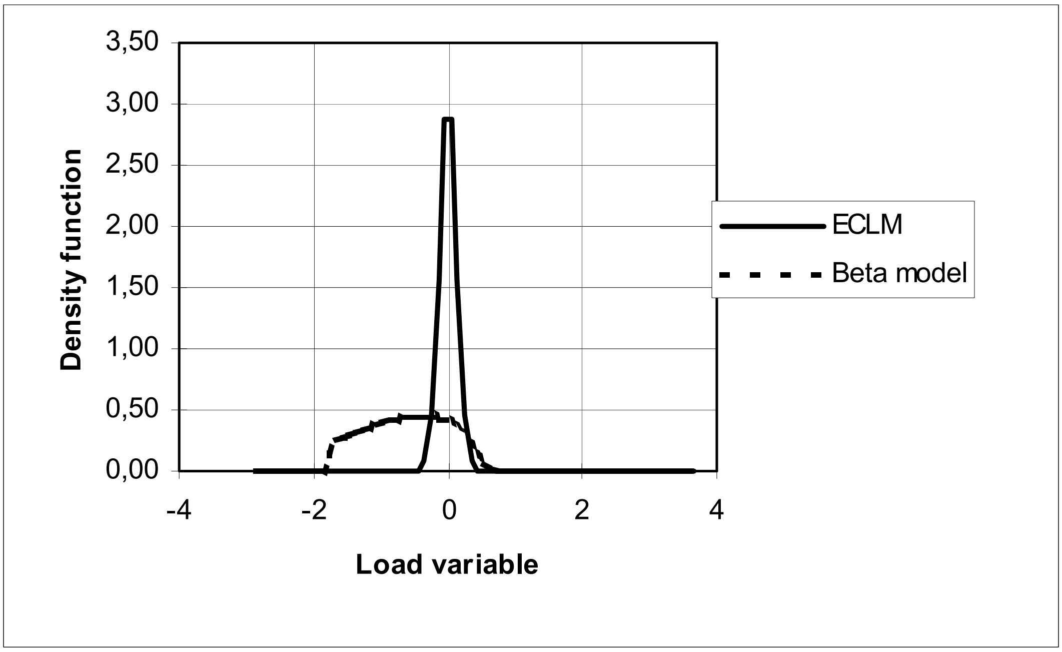

Figure 11 b) Load distributions for data set 2.2. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

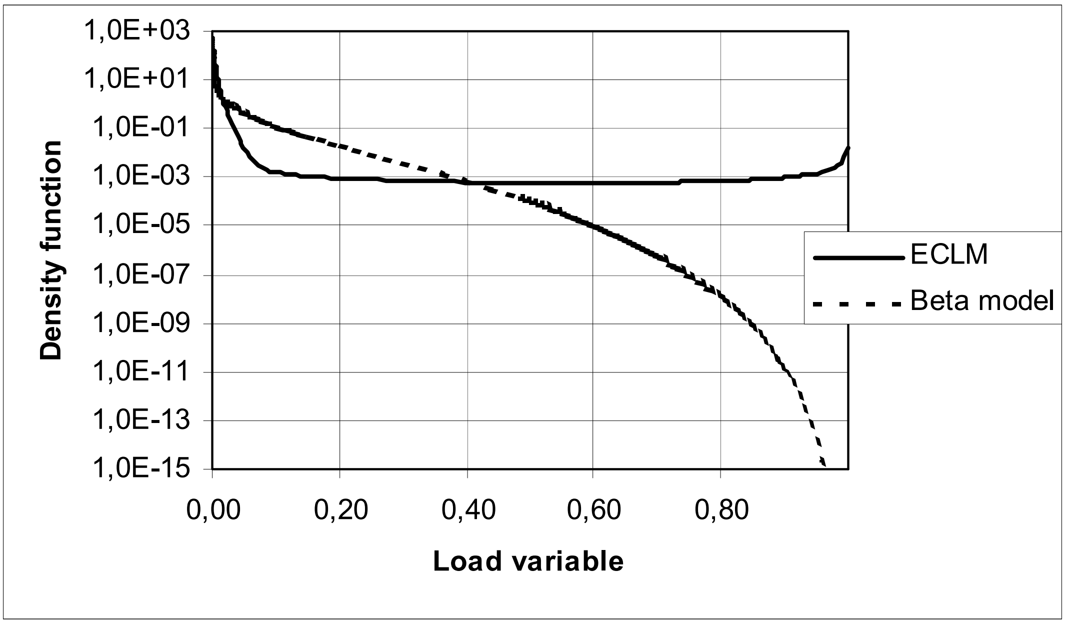

Figure 12 a). Load distributions for data set 2.5. The load distribution of ECLM transformed such that the strength distribution is uniform**.**

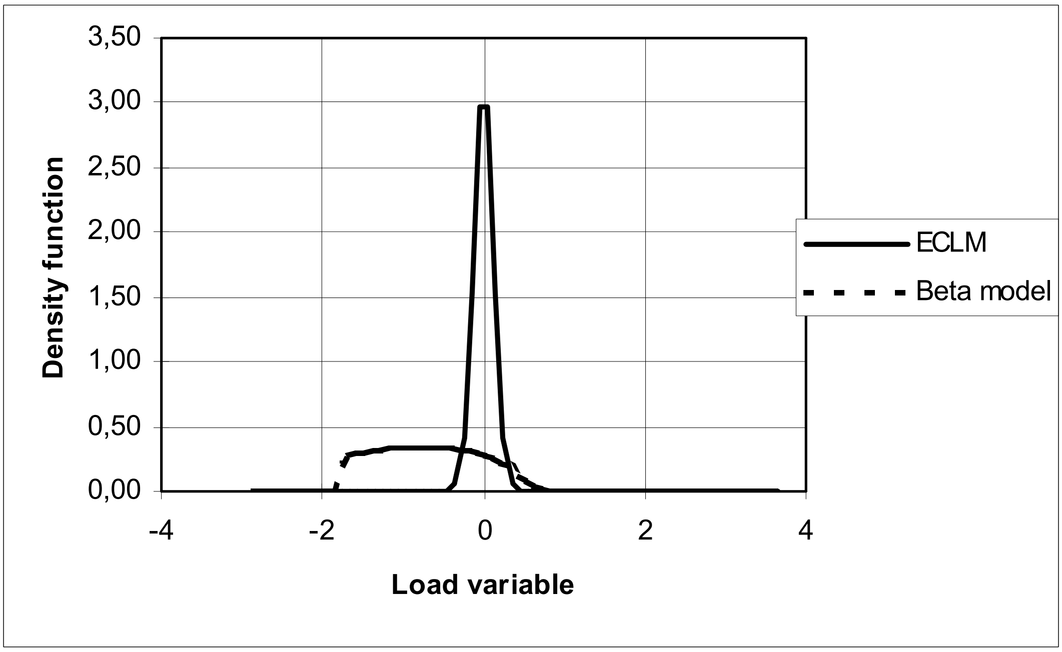

Figure 12 b) Load distributions for data set 2.5. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

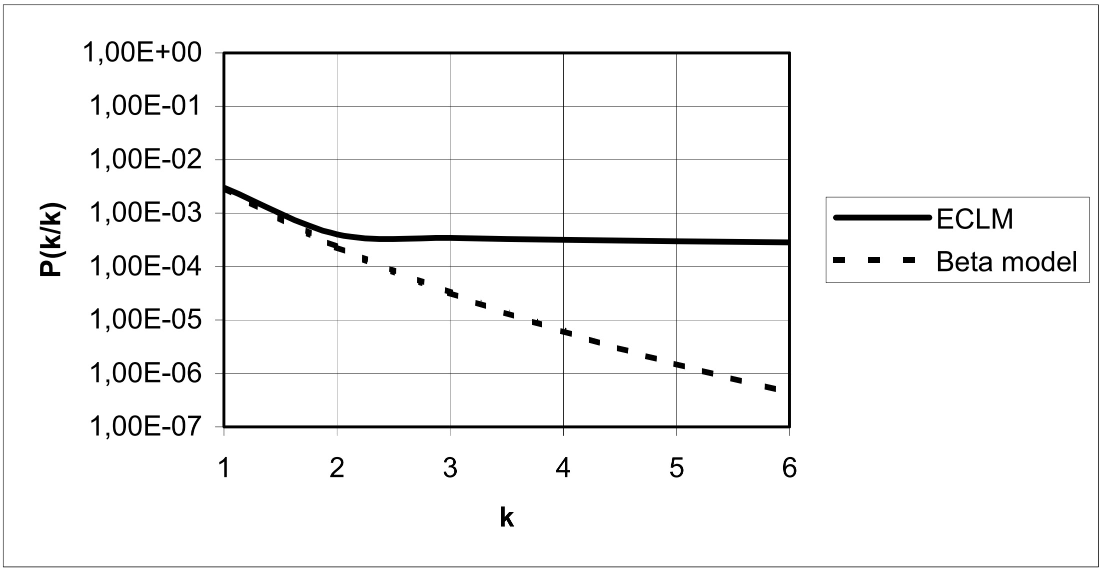

Figure 12. Multiple failure probabilities P(k out of k fail) for data set 2.1.

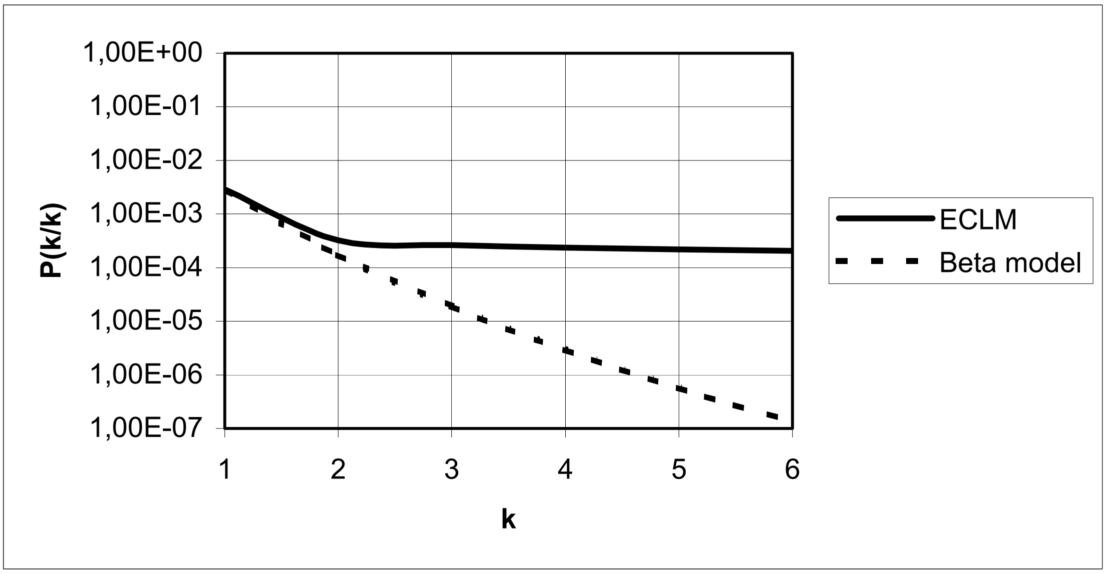

Figure 13. Multiple failure probabilities P(k out of k fail) for data set 2.2.

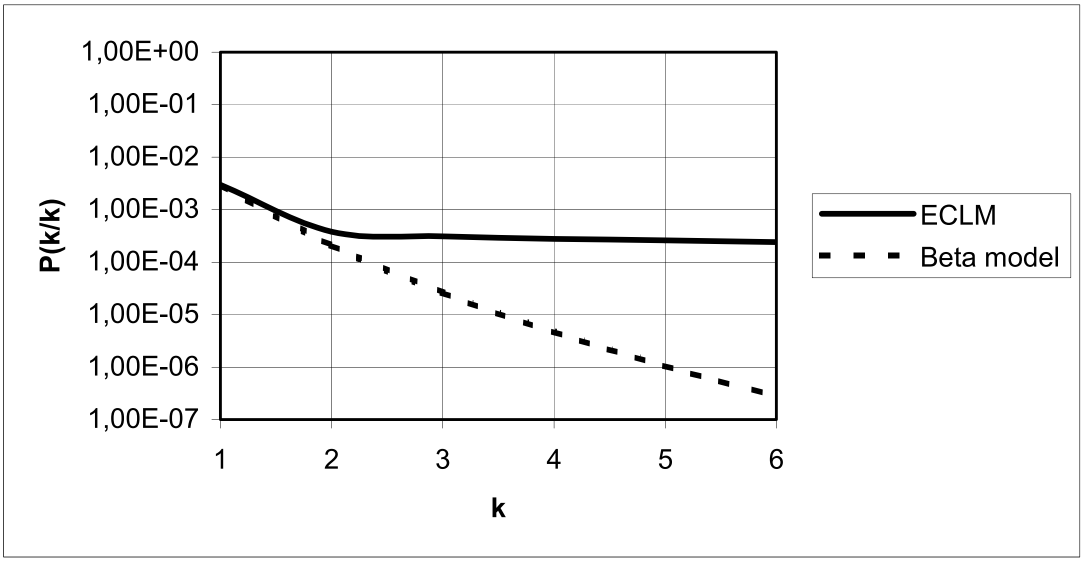

Figure 14. Multiple failure probabilities P(k out of k fail) for data set 2.3.

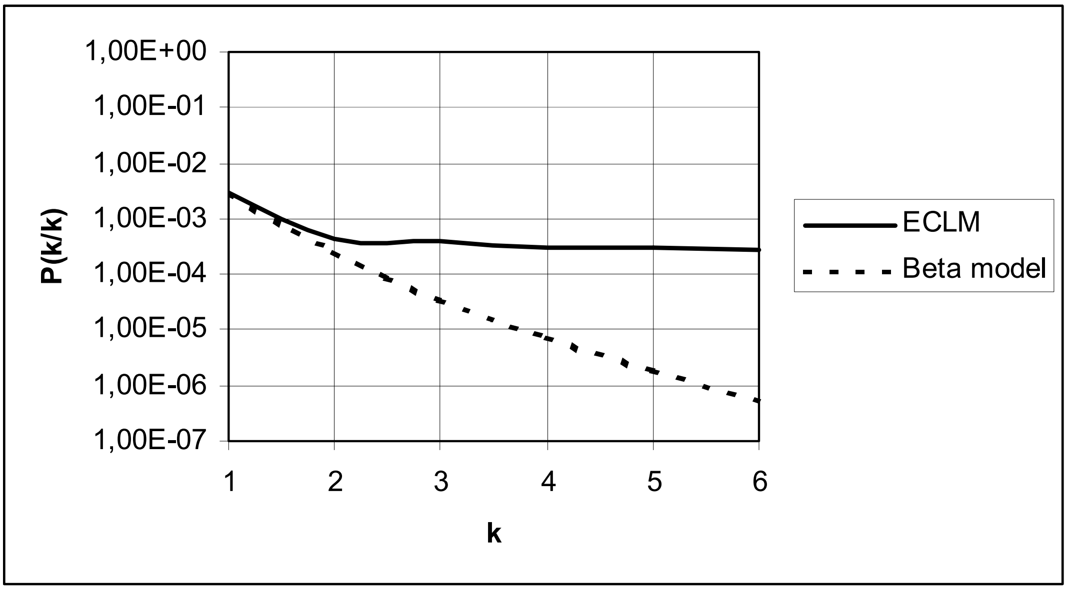

Figure 15. Multiple failure probabilities P(k out of k fail) for data set 2.4.

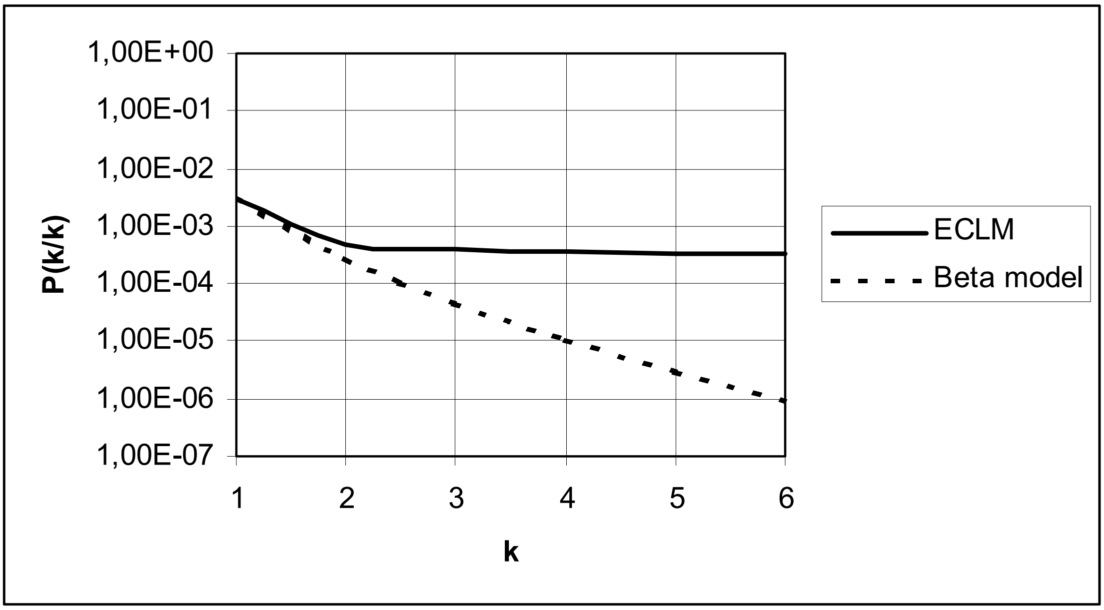

Figure 16. Multiple failure probabilities P(k out of k fail) for data set 2.5.

3.4 Data set 3

The third data set (Table 7) consists of four different variants. The number of system demands is again high (~16067). The evidence can be regarded as very strong (at least for low failure multiplicity). The number of failures of highest multiplicity is now relatively low. The data set 3 differs from the data set 2 in such a way that the number of single and multiple failures is significantly lower.

Table 7. Data set 3

Multiplicity	Data set 3.1	Data set 3.2	Data set 3.3	Data set 3.4
0	15977.8887	15980	15980	15980
1	84.08	84	81	81
2	1.8763	2	3	3
3	0.9866	1	1	1
4	0.4725	0	1	1
5	0.4642	0	1	0
6	0.2317	0	0	1
Total	155453	15451	15452	15453

The estimated parameter values for the Beta model are in the Table 8. Also in this case the ratio of parameters is almost the same for all data set, and the parameter values get smaller with increasing number of failures with highest multiplicity.

Table 8. Parameter estimates of Beta-model for data set 3.

Parameter	Data set 3.1	Data set 3.2	Data set 3.3	Data set 3.4
α	0.025259	0.052533	0.17152	0.015613
β	25.17	55.591	16.611	14.926

The parameter estimates for ECLM are in Table 9. F or this data set, the proportion parameter w_x is approximately same for cases 3.1-3-3, but it is slightly smaller in the case 3.4 (where the number of the six-fold failures is largest). For this data set, the other parameters vary slightly from case to case, and the dependence seem not to be described by single model parameter as in earlier data sets. The parameters for the case 3.4 differ quite a lot from those of the other data cases.

Table 9. Parameter estimates of ECLM for data set 3

Parameter	Data set 3.1	Data set 3.3	Data set 3.3	Data set 3.4
w_b	0.99942	0.99942	0.99913	0.96108
w_x	0.00058	0.00058	0.00087	0.03892
σ_R	0.30096	0.27571	0.28793	0.32269
σ_b	0.10581	0.16343	0.1349	0.28604
σ_x	0.30096	0.27571	0.26045	0.66627
y_x	0.69904	0.72429	0.71207	0.67731

The load distributions are presented for data sets 3.1 and 3.4 (Figures 17 –18). As earlier, the load distribution of Beta model is broader, and it has more probability mass on smaller load values that ECLM.

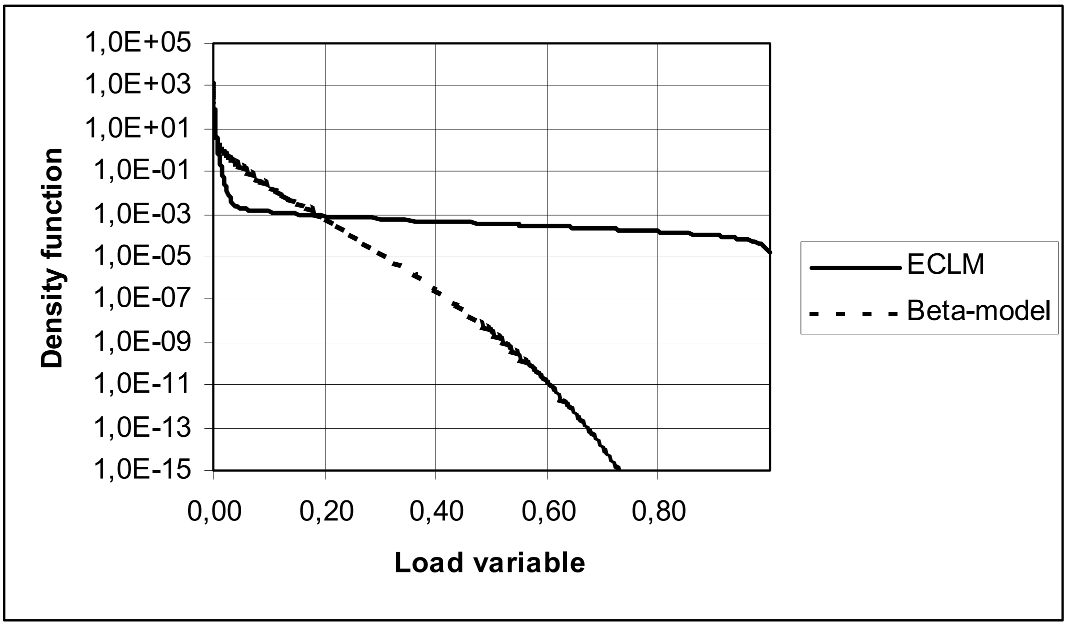

Figure 17 a). Load distributions for data set 3.1. The load distribution of ECLM transformed such that the strength distribution is uniform**.**

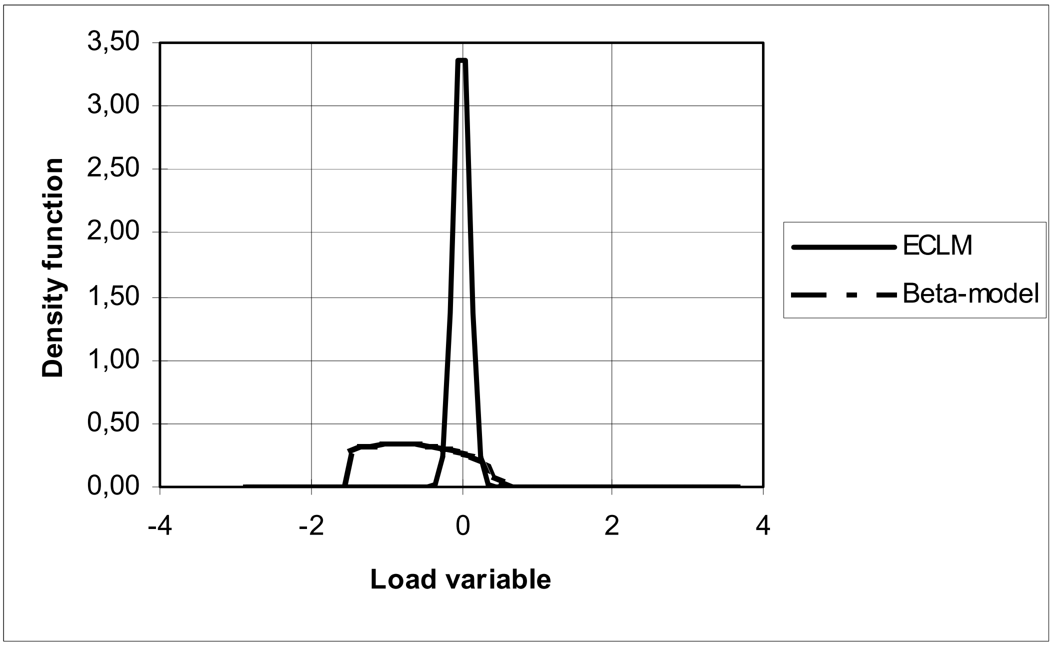

Figure 17b) Load distributions for data set 3.1. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

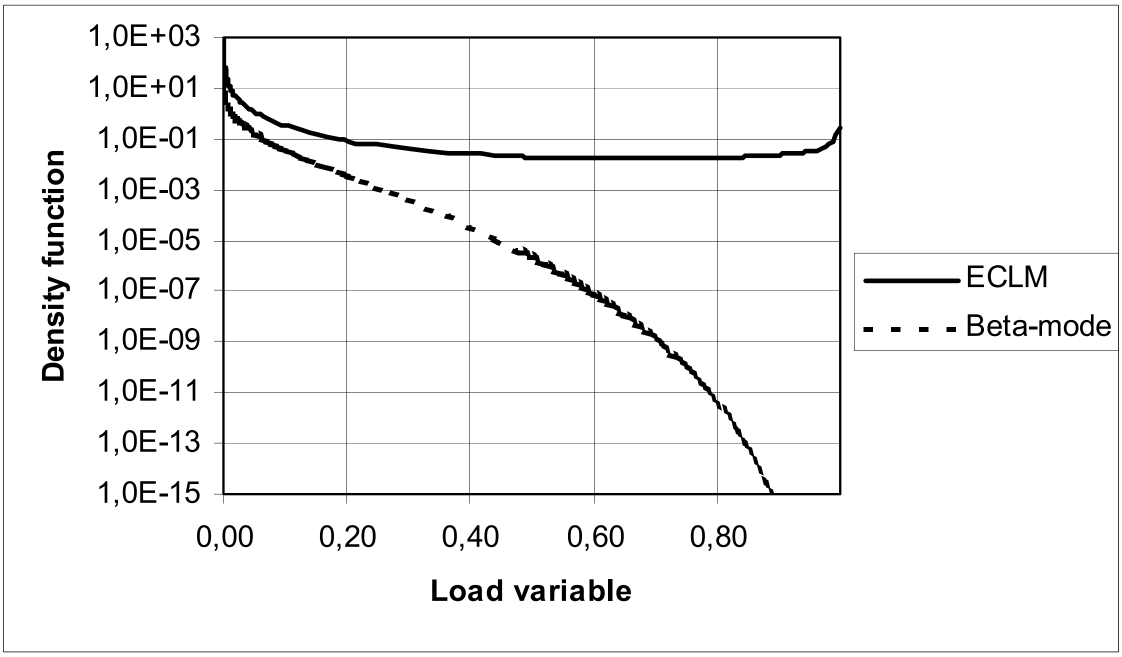

Figure 18 a). Load distributions for data set 3.4. The load distribution of ECLM transformed such that the strength distribution is uniform.

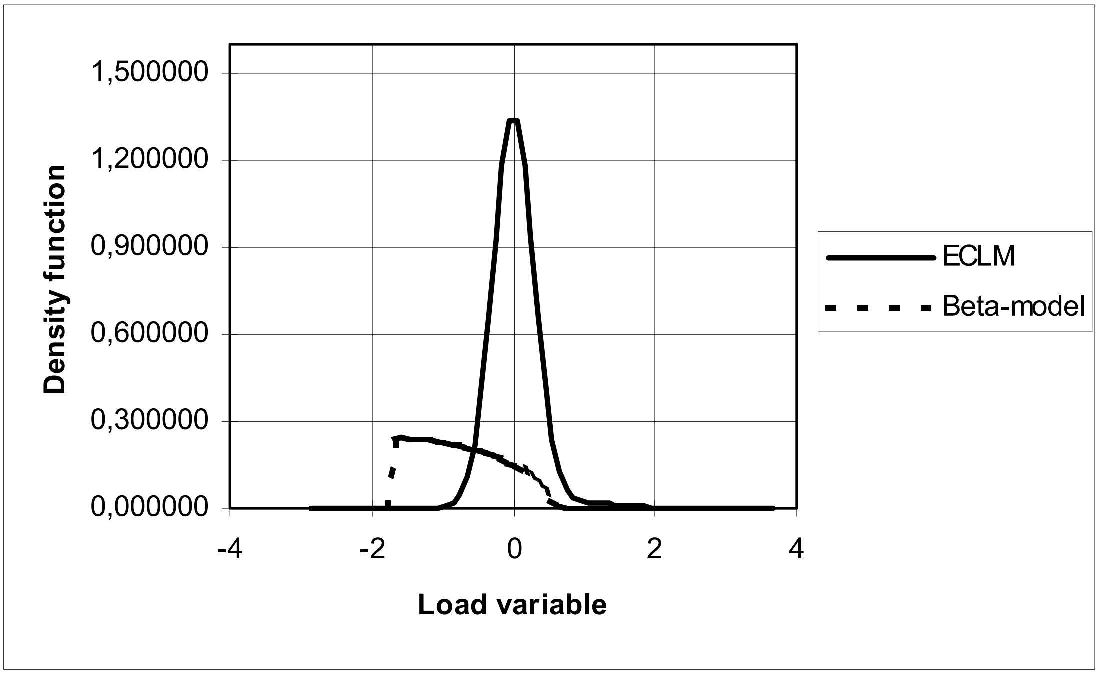

Figure 18b) Load distributions for data set 3.4. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

The probabilities of multiple failures according to the models are presented in Figures 19-22. In all cases, ECLM gives significantly larger multiple failure probabilities. The single and double failure probabilities are rather similar. The predictions of both models are rather sensitive to changes of number of multiple failure probabilities. The largest difference in the probability of 6-fold failure is in the data case 3.2, where the number of high multiplicity failures is smallest.

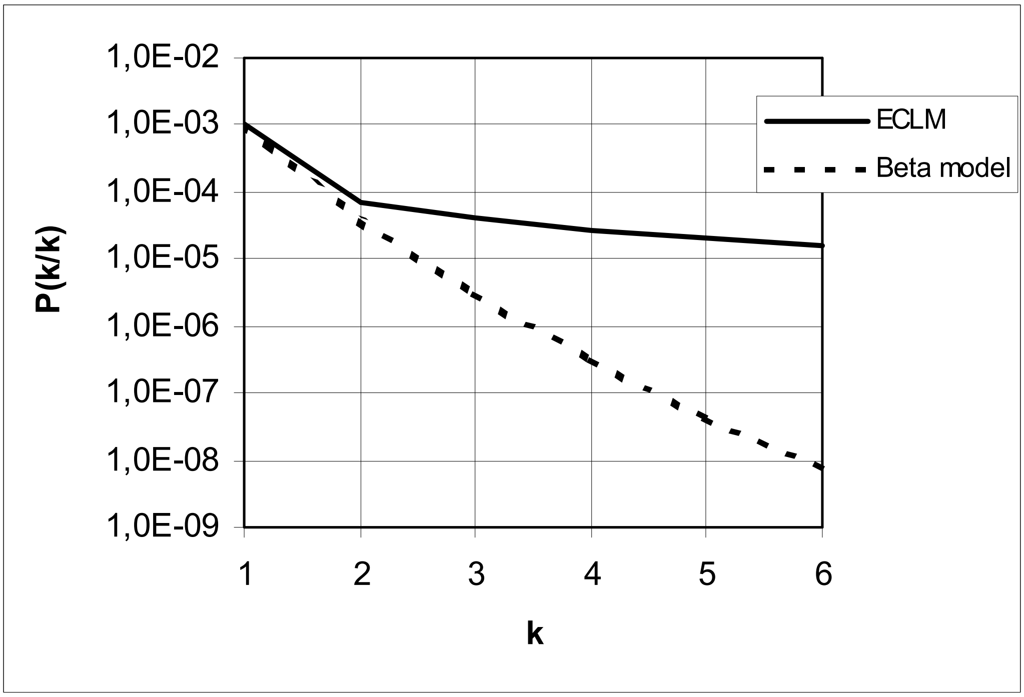

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Figure 19. Multiple failure probabilities P(k out of k fail) for data set 3.1

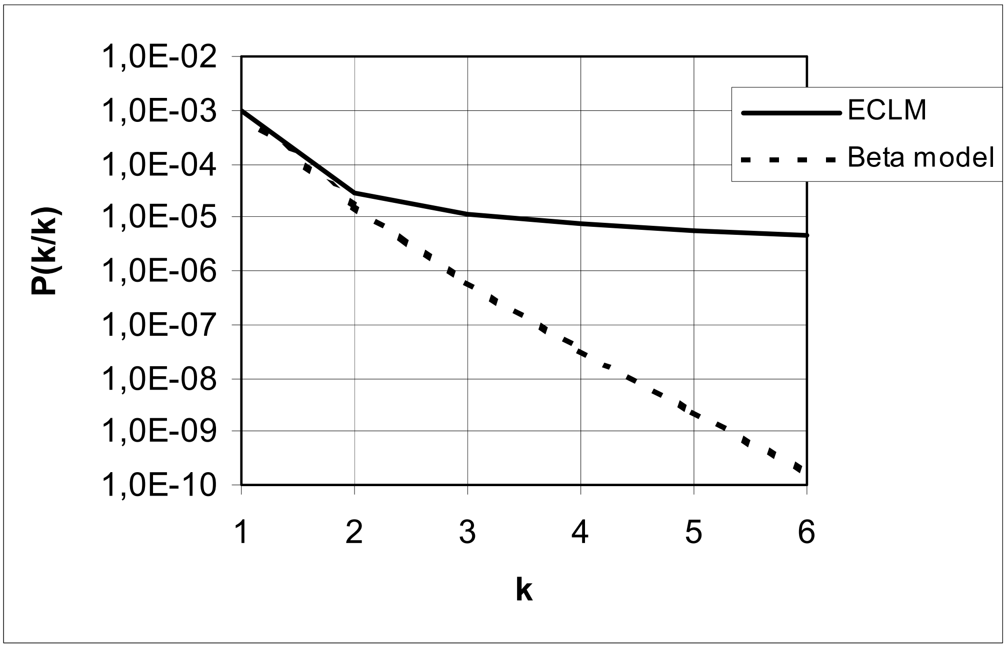

Figure 20. Multiple failure probabilities P(k out of k fail) for data set 3.2.

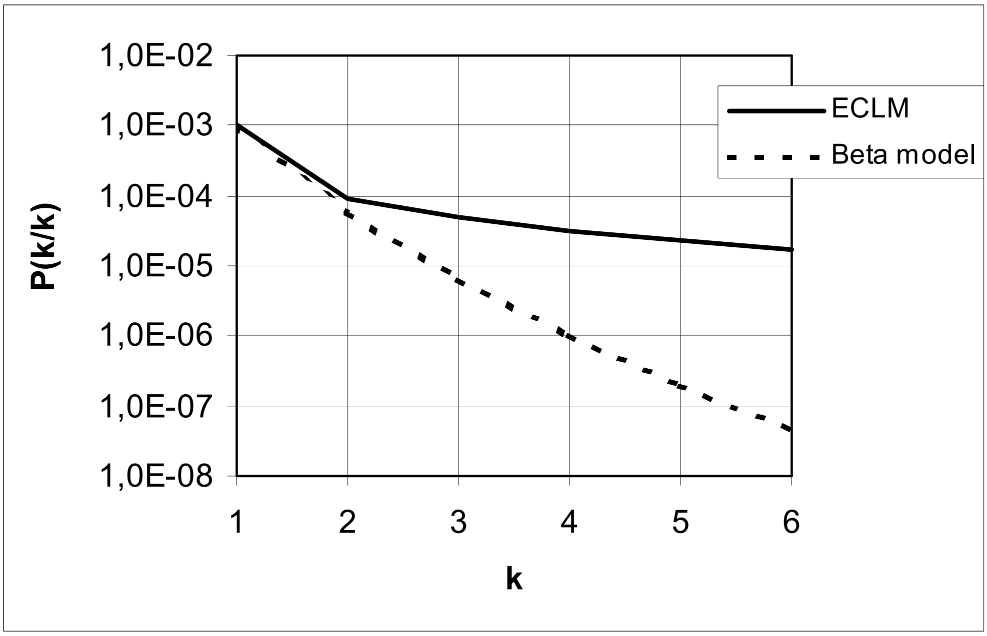

Figure 21. Multiple failure probabilities P(k out of k fail) for data set 3.3.

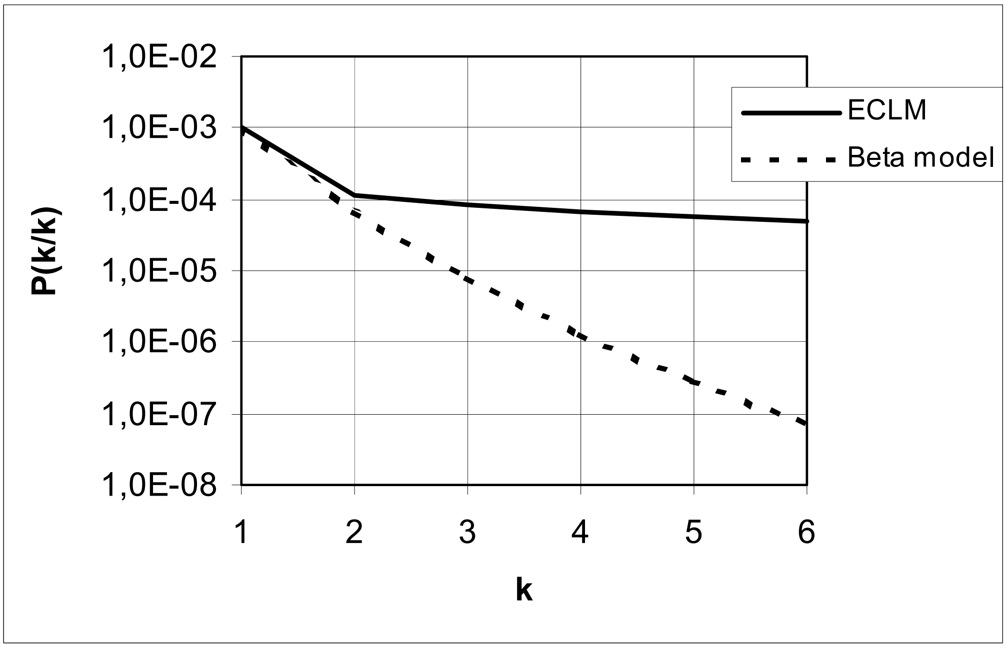

Figure 22. Multiple failure probabilities P(k out of k fail) for data set 3.4.

3.5 Data set 4

The data set 5 consists of failure information from five systems with different degrees of redundancy. The data is in Table 10.

Table 10. The data set 4.

Multiplicity	System 1, number of components 6	System 2, number of components 5	System 3, number of components 4	System 4, number of components 2	System 5, number of components 3
0	200	100	40	100	200
1	5	5	1	6	8
2	2	1	0	2	0
3	3	0	0	-	2
4	0	0	1	-	-
5	0	2	-	-	-
6	0	-	-	-	-
Total	210	108	42	108	210

The parameters of the Beta-model for the data set 4 are: α = *0.28188* and β = *0.87277*. It is worth noticing that these parameters correspond to the load distribution with density function having infinite value at load variable values $x = 0$ and $x = 1$. This is consistent with the rather weak evidence from data set 4. The correspondingf parameter estimates of ECLM are in Table 11.

Table 11. Parameter estimates of ECLM for data set 4.

Parameter	Estimate
w_b	0.96108
w_x	0.03892
σ_R	0.32269
σ_b	0.28604
σ_x	0.66627
y_x	0.67731

The load distributions are presented in Figure 23. Again, the load distribution of Beta-model is concentrated more on the smaller load variable values. However, the difference is smaller than in earlier data cases. In the beta-model, there is also some probability mass at the larger values of the load variable.

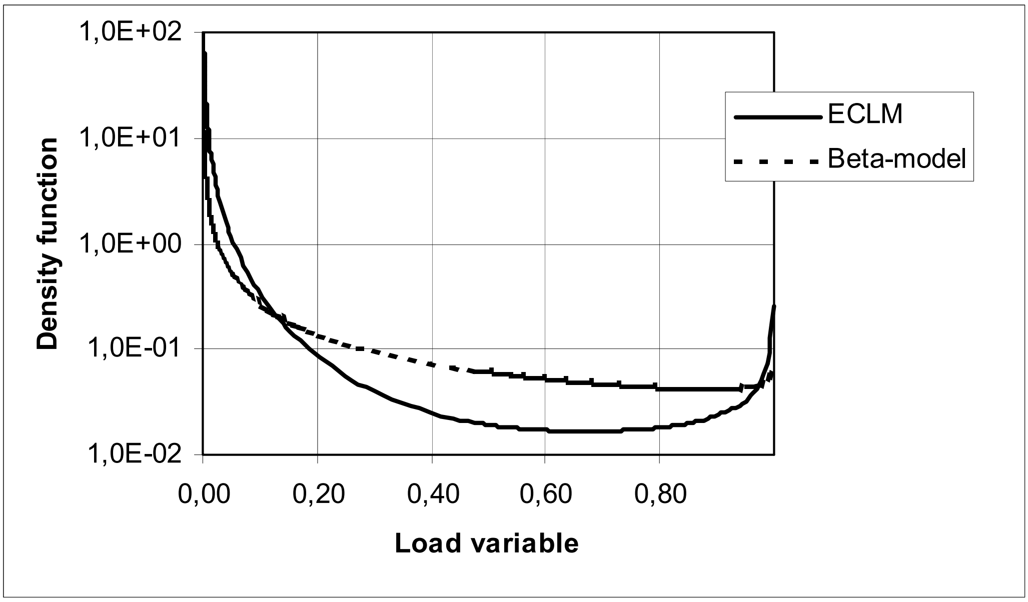

Figure 23 a. Load distributions for data set 4. The load distribution of ECLM transformed such that the strength distribution is uniform.

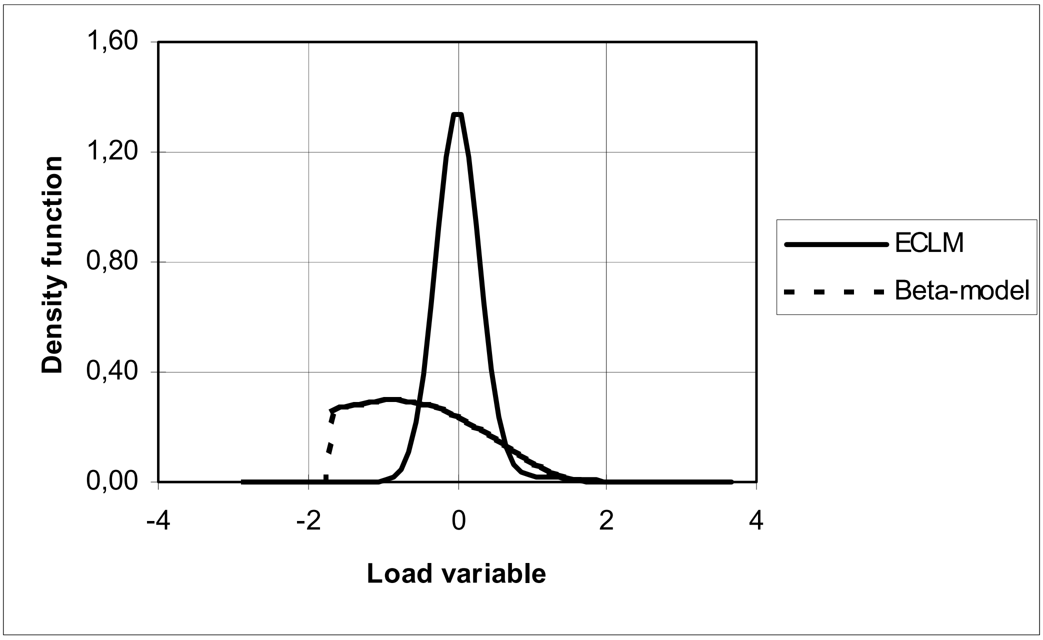

Figure 23 b. Load distributions for data set 4. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

The multiple failures probabilities according to the models are presented in Figures 24. In this data case, the Beta-mode gives higher failure probabilities for all failure multiplicities.

Assuming that the failures are independent and, it is possible to estimate the corresponding single failure probability; $p = 0.0227$. This is compatible with the estimate of ECLM. The corresponding estimate for the Beta-model is $p = 0.0313$, which is significantly larger. This is probably due to the fact that in this data set, the proportion of multiple failures is relatively high, and it has much stronger impact on the estimate of the Beta-model.

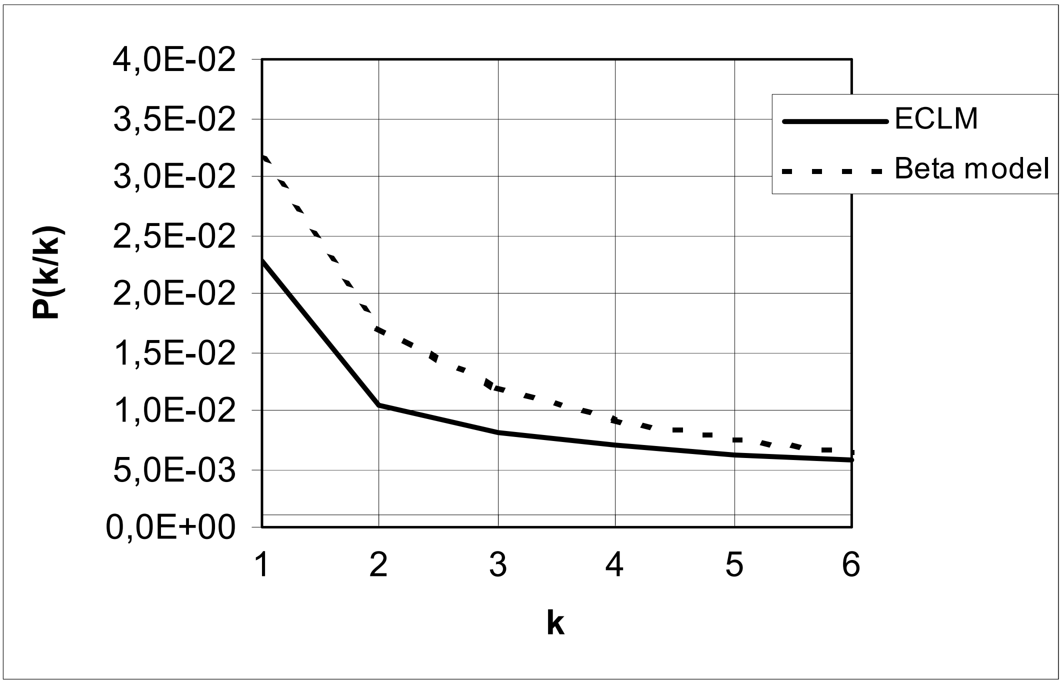

Figure 24. Multiple failure probabilities P(k out of k fail) for data set 4.

3.6 Date set 5

The data set 5 is the CCF data for OKG 314 valves, and it consists of data from two systems with rather high degree of redundancy. This data was used by Alm as background in the development of the Beta-model. It can be seen that the data from two systems are not homogeneous with respect to the number of single and multiple failures. According to Mankamo (2002a) this data doesn't correspond to any realistic data case, and can be used only as a benchmark case for theoretical interests. Mankamo (2002a) states that most of the failures are non-critical with respect to actual demand conditions. The data and the corresponding CCF-estimates should not be used for any practical application, because of strong overestimation of failure probabilities.. The data is in Table 12. The evidence of data set 5 is rather weak, only 110 system demands have occurred, and very few failures (2 single, 1, double and 1 triple failures) have occurred. However, the occurrence of multiple failures seems to indicate dependence between failures. Actually, by assuming independence of failures; the expected number of double (triple, respectively) failures for the system 1 is 0.04 (0.001), which is much less than the occurred number of failures.

Tabl2 12. The data set 5.

Multiplicity	System 1, number of components 13	System 2, number of components 7
0	40	66
1	2	0
2	1	0
3	1	0
4	0	0
5	0	0
6	0	0
7	0	0
8	0	
9	0	
10	0	
11	0	
12	0	-
13	0	
Total	44	66

The parameter estimates fro rhe beta model are: $\alpha = 0.050316$ and $\beta = 8.734$. The parameters of the ECLM are in Table 13.

Table 13. Parameter estimates of ECLM for data set 5.

Parameter	Estimate
w_b	0.99979
w_x	0.00021
σ_R	0.27127
σ_b	0.2999
σ_x	0.54254
y_x	0.72873

The load distributions are presented in Figure 25. Also in this case, the load distribution of the Beta model is more concentrated on smaal values of the load variable. The ECLM load distribution has also stronger tail, which indicates that the multiple failure probabilities should be larger for ECLM.

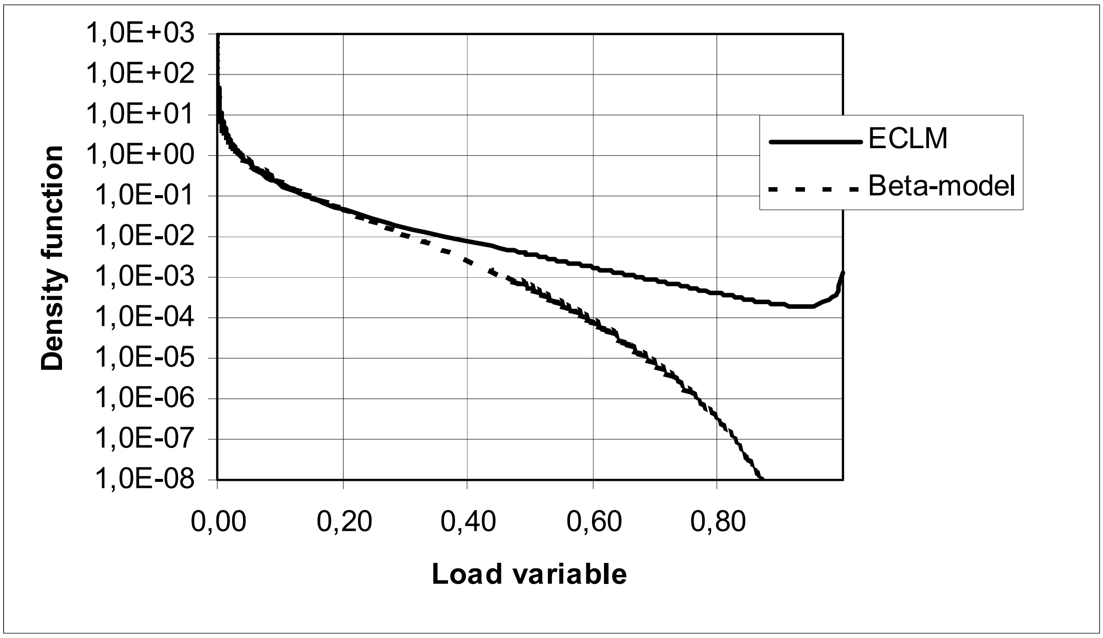

Figure 25a. Load distributions for data set 5. The load distribution of ECLM transformed such that the strength distribution is uniform.

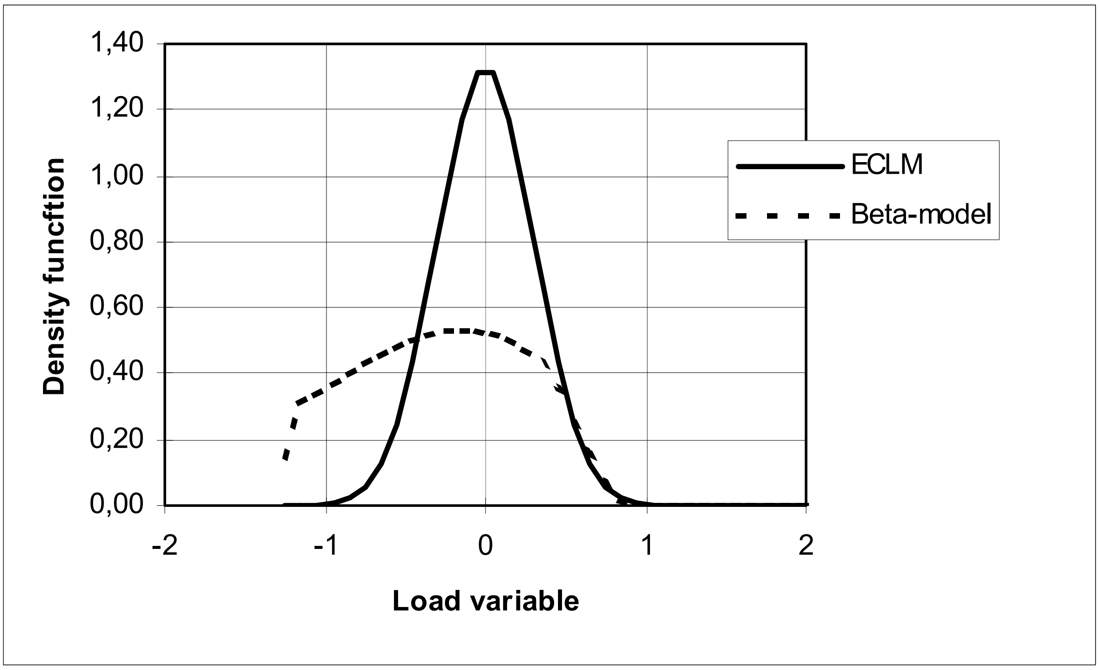

Figure 25 b. Load distributions for data set 5. The load distribution of Beta-model transformed such that the strength distribution is normal with same parameters as ECLM.

The multiple failure probabilities for the models are in Figure 26. In this case the single failure probabilities are rather comparable, but the multiple failure probabilities differ significantly.

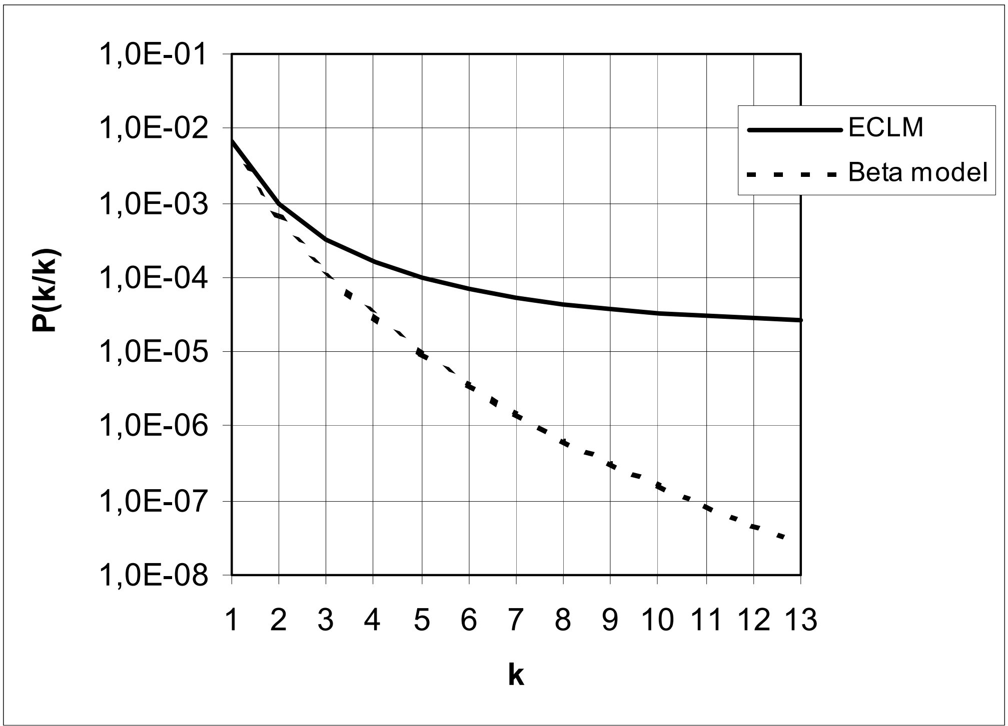

Figure 26. Multiple failure probabilities P(k out of k fail) for data set 5.

3.6 Comparison of likelihoods

As mentioned in section 3.1, the goodness of fit of the models could be compared on the basis of maximal likelihood values: the higher the maximal likelihood, the better is the fit. In the above comparisons, the maximum likelihood value for ECLM is much higher than that of the Beta-model. This is naturally due to the larger number number of parameters in ECLM, which implies better fit to the data.

4 Conclusions

Before going to the conclusions made on the basis of the above experiments, the behaviour of the models will be considered theoretically. In the ECLM model, the dependence between failures is described by several parameters. Depending on the case, the dependence is reflected mainly in the variances of the strength and load variables and the proportion parameter. The expected value of the extreme load part has also an impact on the dependence. In the Beta-model, the single failure probability, *P(1 component out of 1 fails)* has the form

$$P(1 \text{ out of } 1 \text{ fails}) = \frac{\alpha}{\alpha + \beta}.$$
(11)

If the parameter values increase in such a way that the above probability remains constant, the multiple failure probabilities converge to the corresponding independent failure probabilities, or mathematically

$$\alpha = r\alpha_0, \beta = r\beta_0$$
$$P(k \text{ out of } k \text{ fail}) \rightarrow \left(\frac{\alpha_0}{\alpha_0 + \beta_0} \right)^k, \text{ when } r \rightarrow \infty.$$
(12)

This behaviour is demonstrated in Figure 27, where the multiple failure probabilities with different values of *r* are presented. In Figure 27, the parameters $\alpha_0 = 0.050316$ and $\beta_0 = 8.734$ are the estimates corresponding the data set 5.

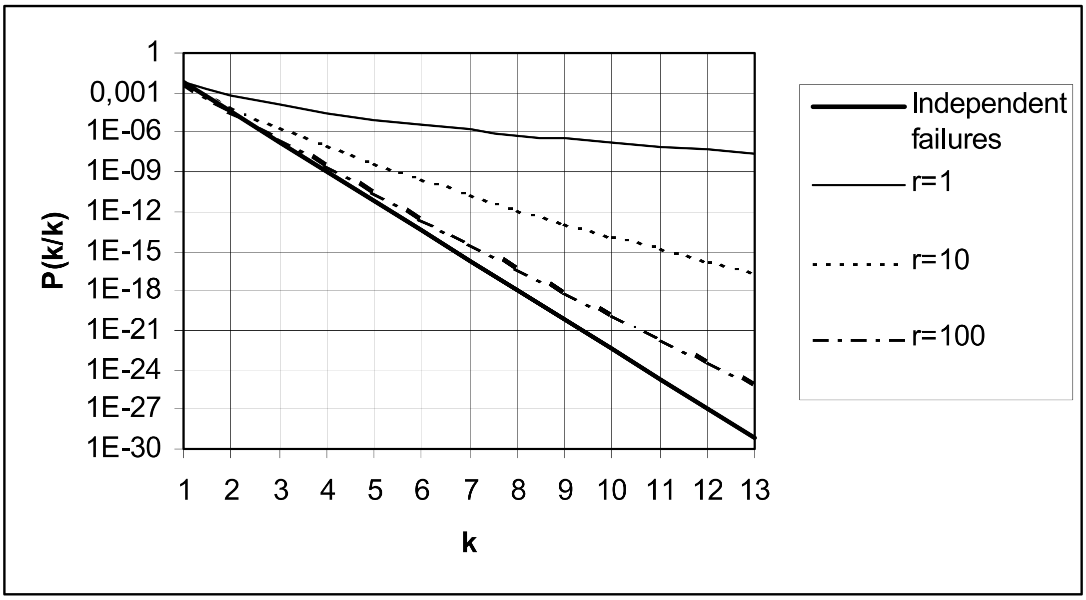

Figure 27. Behaviour of the Beta-mode with increasing parameter values.

This kind of behaviour can be interpreted so the dependence of failures in the Beta-model is described with one parameter (*r*), and the total single failure probability determines the order of the magnitude of the failure probabilities. In this sense, the Beta-model is comparable to the Matti Heikkilä's model (see Mankamo 1992). However, the weak point in Beta-model is that it can only describe dependencies of this type.

In many practical cases, the dependency between failures gets stronger when the failure multiplicity increases, and this kind of behaviour cannot be dealt properly with by Beta-model. The data sets 1-3, which are either real practical cases or their simple modifications are examples of such cases. In the ECLM model, the description of dependence is richer, and it fits very well to the many practical dependency structures.

In fact, if directs estimates for multiple failure probabilities are estimated in the above mentioned data cases, the ECLM predictions are almost exactly the same. From this it can be concluded that when data shows dependency which increases with increasing failure multiplicity, the Beta-model may underestimate the multiple failure probability and it should be used with care. However, for low failure multiplicities Beta-model may give good results.

It is natural that the ECLM "fits better" to the observations, since it has more parameters that the Beta-model. However, in the case of low failure multiplicities, ECLM creates an overfitting, since it has a parameter almost for all failure multiplicity (ECLM is actually developed for describing the CCFs of high multiplicity, and it should not be used without care for other cases). From a statistical modelling point of view, this may not be favourable. The Beta-model is simple and rather easy to interpret, and the small number of parameters is an advantage (at least in the case of low failure multiplicity).

In all cases except one (the data set 4) ECLM gave higher multiple failure probabilities. The differences between the results were significant in almost all cases. As a multi-parameter model, in fits better to the multiple failure data in those cases. Usually, in the failure data, there are many single failures, but the number of actual CCFs is low. In these cases, the Beta-model adjusts its parameters according to the low multiplicity failures mainly, implying rather low dependence. From PSA point of view, this may lead to over-optimistic results. On another hand, ECLM may give more conservative results, since it seems to adjust the dependence according to the failures of highest multiplicity. In any case it clear that the models lead to differences of PSA results.

The data set 4 is an interesting exception. It corresponds to relatively weak evidence compared to the other cases, but the number of multiple failures is high compared to that of single failures. The Beta-model seems to track this rather high dependency by increasing also the single failure probability (see Figure 24).

The sensitivity of parameter estimates to the changes of failure data has been studied in the data sets 1-3. It seems that ECLM parameters are more robust for the changes of data. In the most of the cases it tracks the changes of dependency by changing the variance of the extredme load part. However, in the data set 3.4, also the proportion parameter and the mean of the extreme load part changes more that in the other cases. The Beta-model parameters change rather much along the changes in data, but the single failure probability is relatively stable.

Both models are estimated by using the maximum likelihood principle. However, the confidence regions for the parameter are not determined. It would be rather simple task to determine approximate confidence bounds for the estimates by using the Fisher's information. Another possibility is to apply Bayesian methods. The determination of confidence bounds is recommended for both of the models.

In this comparison study, the models were compared with a limited number of data cases. It was not possible to make the comparison according to the best statistical principles and theoretical analyses. One possibility to make such a comparison is to simulate a large number of data cases from some CCF-model (with known multpile failure probabilities) and to estimate the parameters of the ECLM and Beta-model on the basis of the simulated data. In such a comparison, the statistical properties (e.g. the goodness of fit, the consistency of estimates, sensitivity of the parameters etc.) could have been studied. This comparison gives only a view on some rather evident the properties of the models.

It must be emphasised that the most important issue in CCF-analysis is not the models, but the amount and quality of the raw data. The treatment of the data is mainly engineering work and it is at least partially independent on the applied CCF-model. Another question is how well the applied model supports the use of expert judgements, but it was not studied here.

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Abstract

The report describes a simple comparison of two CCF-models, the ECLM, and the Beta-model. The objective of the comparison is to identify differences in the results of the models by applying the models in some simple test data cases. The comparison focuses mainly on theoretical aspects of the above mentioned CCF-models. The properties of the model parameter estimates in the data cases is also discussed. The practical aspects in using and estimating CCF-models in real PSA context (e.g. the data interpretation, properties of computer tools, the model documentation) are not discussed in the report. Similarly, the qualitative CCF-analyses needed in using the models are not discussed in the report.

Key words Comparison, CCF-models, ECLM, Beta-model

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